

HOMWORK 2 SOLUTION

Problem 1:

1.) Monthly payment?

$$n = 25 \times 12 \text{ or } 300$$

$$i = 6\% \text{ per year}$$

$$PV = \$100,000$$

$$FV = 0$$

$$\text{Solve for payment: } PMT = \$644.30$$

2.) Month 1:

$$\text{Interest payment: } \$100,000 \times (6\%/12) = \$500$$

$$\text{Principal payment: } \$644.30 - \$500 = \$144.30$$

3.) Entire 25 Year Period:

$$\text{Total payments: } \$644.30 \times 300 = \$193,290.42$$

$$\text{Total principal payment: } \$100,000$$

$$\text{Total interest payments: } \$193,290.42 - \$100,000 = \$93,290.42$$

4.) Outstanding loan balance if repaid at end of 5 years = \$89,932.09, as presented below using FV method.

$$n = 60 \text{ (pay off period)}$$

$$i = 6\% \text{ per year}$$

$$PMT = \$644.30$$

$$PV = \$100,000$$

$$\text{Solve for FV: } FV = \$89,932.09$$

5.) Through 5 years:

$$\text{Total payments: } \$644.30 \times 60 = \$38,658.08$$

$$\text{Total principal payment (principal reduction): } \$100,000 - \$89,932.09 = \$10,067.91$$

$$\text{Total interest payment: } \$38,658.08 - \$10,067.91 = \$28,590.17$$

6.) Step 1, Solve for loan balance at the end of month 59 using PV method:

$$n = 300 - 59 = 241$$

$$i = 6\% \text{ per year}$$

$$PMT = \$644.30$$

$$FV = 0$$

$$\text{Solve for loan balance: } PV = \$90,125.76$$

Step 2, Solve for the interest payment at month 60:

$$\text{Interest payment: } \$90,125.76 \times (.06/12) = \$450.63$$

$$\text{Principal payment: } \$644.30 - \$450.63 = \$193.67$$

Problem 2:

1.) Monthly Payment PMT?

$$\begin{aligned}n &= 20 \times 12 = 240 \\i &= 5\% \text{ per year} \\PV &= \$150,000 \\FV &= 0\end{aligned}$$

$$\text{Solve for payment: } PMT = \$989.93$$

2.) Outstanding loan balance if repaid at end of year 10?

$$\begin{aligned}n &= 120 \\i &= 5\% \text{ per year} \\PMT &= \$989.93 \\PV &= \$150,000\end{aligned}$$

$$\text{Solve for mortgage balance: } FV = \$93,332.28$$

$$\begin{aligned}\text{Total interest collected} &= \text{Total payment} - (\text{loan amount} - \text{mortgage balance}) \\&= \$989.93 \times 120 - \$150,000 + \$93,332.28 \\&= \$62,123.88\end{aligned}$$

3.) Loan balance reduced by \$20,000 at the end of year 10:

$$\text{Remaining balance} = \$93,332.28 - \$20,000 = \$73,332.28$$

The new loan maturity if payment not reduced?

$$\begin{aligned}i &= 5\% \text{ per year} \\PMT &= -\$989.93 \\PV &= \$73,332.28 \\FV &= 0\end{aligned}$$

$$\text{Solve for maturity: } n = 88.8 \approx 89 \text{ months}$$

Problem 3:

1) Annual Payments?

$$\begin{aligned}n &= 10 \\i &= 8\% \text{ per year} \\PV &= -\$3,000,000 \\FV &= \$1,500,000\end{aligned}$$

$$\text{Solve for annual payment: } PMT = \$343,544.23$$

2) Loan balance at the end of year 5?

$$\begin{aligned}n &= 5 \\i &= 8\% \\PMT &= \$343,544.23 \\FV &= \$1,500,000\end{aligned}$$

Solve for the loan balance: $PV = -\$2,392,547.30$

Total payment at end year 5 = Year 5 payment + loan balance
 $= \$2,392,547.30 + \$343,544.23 = \$2,736,091.53$

Problem 4:

1.) Loan amount = $\$300,000 \times 80\% = \$240,000$

Amount disbursed by the lender = $\$240,000 - \$5,000 = \$235,000$

2.) Monthly payments?

$n = 360$
 $i = 6\%$ per year
 $FV = 0$
 $PV = \$240,000$

Monthly Payment $PMT = \$1,438.92$

The effective interest rate if no prepayment?

$n = 360$
 $PMT = \$1,438.92$
 $FV = 0$
 $PV = \$235,000$

Effective Interest rate = 6.2%

3.) Assuming the loan payoff occurs after 5 years, determine the mortgage balance:

$n = 300$
 $PMT = \$1,438.92$
 $FV = 0$
 $I = 6\%$ per annum

Loan balance at end of year 5 is: $PV = \$223,330.46$

The effective interest rate would be:

$n = 60$
 $PMT = \$1,438.92$
 $PV = -\$235,000$
 $FV = \$223,330.46$

Effective interest rate = 6.50%

This effective interest rate is higher because the loan origination fee is amortized over a much shorter period (5 years instead of 30 years).

4.) With a prepayment penalty of 2% on the outstanding loan balance at the end year 10.

$n = 120$
 $PMT = \$1,438.92$
 $PV = -\$240,000$

$I = 6\%$ per annum

Loan balance at end of year 10 is: $FV = \$200,845.74$

The effective interest cost would be:

$n = 120$

$PMT = \$1,438.92$

$PV = -\$235,000$

$FV = \$200,845.74 \times 1.02 =$

Effective interest rate $i = 6.43\%$

Problem 5:

1.) Monthly Payments to the borrower?

$n = 120$

$i = 8\%$ per annum

$PV = 0$

$FV = \$400,000$

Solve for monthly payments: $PMT = \$2,186.44$

2.) Balance at end year 5 if borrower receives \$2,186.44 per month

$n = 60$

$i = 8\%$

$PV = 0$

$PMT = \$2,186.44$

Solve for loan balance: $FV = \$160,652.53$

3.) The borrower will receive \$3,000 per month for 60 months and then the maximum amount he can receive from months 61 to 120 is:

$n = 60$

$i = 8\%$

$PV = 0$

$PMT = \$3,000$

Solve for loan balance: $FV = \$220,430.57$

Step 2, Solve for payments during months 61 to 120:

$n = 120 - 60 = 60$

$i = 8\%$ per annum

$PV = -\$220,430.57$

$FV = \$400,000$

Max monthly payments 611 through 120: $PMT = \$974.35$

Problem 6

1.) Calculate constant monthly amortization: $\$200,000 \div 180 \text{ months} = \$1,111.11$ per month

Month	Beg. Balance	Interest	Amortization	Total Payment	End Balance
1	200,000.00	2,000.00	1111.11	3,111.11	198,888.89
2	198,888.89	1,988.89	1111.11	3,100.00	197,777.78
3	197,777.78	1,977.78	1111.11	3,088.89	196,666.67
4	196,666.67	1,966.67	1111.11	3,077.78	195,555.56
5	195,555.56	1,955.56	1111.11	3,066.67	194,444.44
6	194,444.44	1,944.44	1111.11	3,055.56	193,333.33

2.) For a constant payment loan (CPM) we have:

PV = \$200,000

n = 180

i = 12% per year

FV = 0

Solve PMT = \$2,400.34

Month	Beg. Balance	Total Payment	Interest	Amortization	End Balance
1	200,000.00	2,400.34	2,000.00	400.34	199,599.66
2	199,599.66	2,400.34	1,996.00	404.34	199,195.32
3	199,195.32	2,400.34	1,991.95	408.38	198,786.94
4	198,786.94	2,400.34	1,987.87	412.47	198,374.47
5	198,374.47	2,400.34	1,983.74	416.59	197,957.88
6	197,957.88	2,400.34	1,979.58	420.76	197,537.13

3.) During the first year of the loan, a CAM would be less risky for the lender because it pays down the principal faster.

4.) In the absence of point and origination fees, the effective interest rates on both loans will be an annual rate of 12%, compounded monthly. This is true regardless of when either of the loans are repaid. Monthly payments are different, however i is the same for both loans.