

SOLUTIONS HOMEWORK 1

Problem 1

a)	Future Value	=	FV(n,i,PV,PMT)
		=	FV (7yrs, 6%, \$12,000, 0)
		=	\$18,044 (annual compounding)
b)	Future Value	=	FV(n,i,PV,PMT)
		=	FV (28 quarters, 9% ÷ 4, \$12,000, 0)
		=	\$22,375 (quarterly compounding)
c) Equivalent annual yield: (consider one year only)			
	Future Value of (a)	=	FV(n,i,PV,PMT)
		=	FV (1yr, 6%, \$12,000, 0)
		=	\$12,720
	(\$12,720 - \$12,000) / \$12,000 =		6.00% effective annual yield
	Future Value of (b)	=	FV(n,i,PV,PMT)
		=	FV (1yr, 9%, \$12,000, 0)
		=	\$13,117
	(\$13,117 - \$12,000) / \$12,000 =		9.31% effective annual yield

Alternative (b) is better because of its higher effective annual yield.

Problem 2

Find the future value of 24 deposits of \$5,000 made at the end of each 6 months. Deposits will earn an annual rate of 8.0%, compounded semi-annually.

Future Value	=	FV(n,i,PV,PMT)
	=	FV (24 periods, 8% ÷ 2, 0, \$5,000)
	=	\$195,413

Note: Total cash deposits are \$5,000 x 24 = \$120,000. Total interest equals \$75,413 or (\$195,413 - \$120,000). The \$120,000 represents the return of capital (initial principal) while the \$75,413 represents the interest earned on the capital contributions.

Find the future value of 24 *beginning-of-period* payments of \$5,000 at an annual rate of 8.0%, compounded semi-annually based on an annuity due.

Future Value	=	FV(n,i,PV,PMT)
	=	FV (25 periods, 8% ÷ 2, 0, \$5,000) -5000
	=	\$203,230

Note: n is changed to 25 because the deposits are made at the beginning of each period and we can basically assume that payments are made at the end of the periods.

Problem 3

End of Year	Amount Deposited	FV(n,i,PV,PMT)	Future Value
1	\$2,500	FV(4 yrs, 15%, \$2,500, 0)	\$4,372.52
2	\$0	FV(3 yrs, 15%, 0, 0)	\$0.00
3	\$750	FV(2 yrs, 15%, \$750, 0)	\$991.88
4	\$1,300	FV(1 yr, 15%, \$1,300, 0)	\$1,495.00
5	\$0		\$0.00
Total Future Value =			<u>\$6,859.40</u>

The investor will have \$6,859.40 on deposit at the end of the 5th year.

*Each deposit is made at the end of the year.

Problem 4

Find the present value of 10 end-of-year payments of \$2,150 discounted at an annual interest rate of 12 percent.

$$\begin{aligned} \text{Present Value} &= \text{PV (n,i,PMT,FV) - ordinary annuity} \\ &= \text{PV (10 yrs, 12\%, \$2,150, 0)} \\ &= \$12,148 \text{ should be paid today} \end{aligned}$$

Find the present value of 10 beginning-of-year payments of \$2,150 discounted at an annual interest rate of 12 percent.

$$\begin{aligned} \text{Present Value} &= \text{PV (n,i,PMT,FV)} \\ &= \text{PV (9 yrs, 12\%, \$2,150, 0) + \$2,150} \\ &= \$13,606 \text{ should be paid today} \end{aligned}$$

Note: 1st payment of \$2,150 is not discounted because it is received immediately or at the beginning of year 1. The remaining 9 payments are discounted at 12% annually. This problem illustrates an annuity due.

Problem 5

Find the present value of \$15,000 discounted at an annual rate of 8% for 10 years.

$$\begin{aligned} \text{Present Value} &= \text{PV (n,i,PMT,FV)} \\ &= \text{PV (10 yrs, 8\%, 0, \$15,000)} \\ &= \$6,948 \text{ (annual compounding)} \end{aligned}$$

The investor should not purchase the lot because the present value of the lot (discounted at the appropriate interest rate) is less than the current asking price of \$7,000.

$$\begin{aligned} \text{Interest/IRR} &= i(n,PV,PMT,FV) \\ \text{Interest/IRR} &= i(10\text{yrs, }-\$7,000, \$15,000, 0) \\ \text{Interest/IRR} &= \mathbf{7.92\%} \end{aligned}$$

Problem 6

What will be the rate of return (yield) on a project that initially costs \$100,000 and is expected to pay out \$15,000 per year for the next ten years?

$$\begin{aligned} \text{Interest/IRR} &= i(n,PV,PMT,FV) \\ \text{Interest/IRR} &= i(10 \text{ yrs, }-\$100,000, \$15,000, 0) \end{aligned}$$

$$\text{Interest/IRR} = 8.14\%$$

It is a good investment for DDC because the IRR of 8.14% exceeds DDC's desired return of 8%.

Problem 7

What will be the internal rate of return (yield) on a project that initially costs \$100,000 and is expected to receive \$1,600 per month for the next 5 years and, at the end of the five years, return the initial investment of \$100,000?

$$\begin{aligned} \text{Interest/IRR} &= i(n, PV, PMT, FV) \\ \text{Interest/IRR} &= i(60 \text{ months}, -\$100,000, \$1,600, \$100,000) \\ \text{Interest/IRR} &= 1.6\% \\ &\text{--and } 1.6\% \times 12 = 19.2\% \text{ (internal rate of return of 19.2\%} \\ &\text{compounded monthly)} \end{aligned}$$

Problem 8

Annual sinking fund payments required to accumulate \$60,000 after ten years

$$\begin{aligned} \text{Payment} &= \text{Payment}(n, i, PV, FV) \\ \text{Payment} &= \text{Payment}(10 \text{ yrs}, 10\%, 0, \$60,000) \\ &= \$3,765 \text{ per year} \end{aligned}$$

Note to Instructor: In problem 3-15(b), the text indicates that annual payments be calculated. However, the text should read: monthly payments.

Monthly sinking fund payments required to accumulate \$60,000 after ten years.

$$\begin{aligned} \text{Payment} &= \text{Payment}(n, i, PV, FV) \\ \text{Payment} &= \text{Payment}(120 \text{ periods}, 10\%/12, 0, \$60,000) \\ &= \$292.90 \text{ per month} \end{aligned}$$

$$\text{Annual payments} = 292.90 \times 12 = \$3,514.85$$

Problem 9

a) Find the ENAR for 12% EAY given Monthly Compounding.

$$\begin{aligned} \text{ENAR} &= [(1 + \text{EAY})^{(1/m)} - 1] \times m \\ &= [(1 + .12)^{(1/12)} - 1] \times 12 \\ &= [1.0094888 - 1] \times 12 \\ &= .11387 \text{ or } 11.39\% \end{aligned}$$

b) Find the ENAR for 12% EAY given Quarterly Compounding

$$\begin{aligned} \text{ENAR} &= [(1 + \text{EAY})^{(1/m)} - 1] \times m \\ &= [(1 + .12)^{(1/4)} - 1] \times 4 \\ &= [1.0287373 - 1] \times 4 \\ &= .114949 \text{ or } 11.49\% \end{aligned}$$

Problem 10

Goal: To show the relationship between IRRs, compound interest, recovery of capital and cash flows.

a) Note: the sum of all cash flows is \$17,863.65. The investment is \$13,000, therefore \$4,863.65 must be interest (profit). The goal is (1) to determine the annual breakdown between interest (profit), recovery of capital (principal) from the cash flows and (2) show that compound interest is being earned on the investment balance at an interest rate equal to the IRR. This exercise should prove that the IRR is equivalent to an interest rate of 10% compounded annually. It should also demonstrate the equivalence between an IRR and compound interest.

(b) IRR = 10% (annual rate, compounded annually)

(c) Proof:

<u>Beginning of Year</u>	<u>Investment</u>	<u>10% Interest</u>	<u>Cash Flow</u>	<u>Recovery of Capital (ROC)</u>	<u>End of Year (Balance)</u>
1	13,000.00	\$1,300.00	\$ 5,000.00	\$ 3,700.00	\$ 9,300.00
2	9,300.00	930.00	1,000.00	70.00	9,230.00
3	9,230.00	923.00	-0-	-0-	10,153.00*
4	10,153.00	1,015.30	5,000.00	3,984.70	6,168.30
5	6,168.30	616.83	6,000.00	5,383.17	785.13
6	785.13	78.51	863.65	785.14	-0-
		<u>\$4,863.65</u>	<u>\$17,863.76</u>	<u>\$13,923.00</u>	

* Note: Because the cash flow in year 3 is zero, interest must be accrued on the balance of \$9,230 during year 3 and added to the investment balance.