Atmosphere, Ocean and Climate Dynamics
Answers to Chapter 12

1. *Consider a homogeneous slab of material with a vertical diffusivity, k_v, subject to a flux of heat through its upper surface which oscillates at frequency ω given by $Q_{\text{net}} = \text{Re}\,\widehat{Q}_\omega e^{i\omega t}$ where $\widehat{Q}_\omega$ sets the amplitude of the net heat flux at the surface. Solve the following diffusion equation for temperature variations within the slab,*

$$\frac{dT}{dt} = k_v \frac{d^2T}{dz^2}$$

assuming that $k_v \frac{dT}{dz} = \frac{Q_{\text{net}}}{\rho c}$ at the surface ($z = 0$, where ρ is the density of the material and c is its specific heat) and that $T \longrightarrow 0$ at great depth ($z = -\infty$).

   (a) *Use your solution to show that temperature fluctuations at the surface have a magnitude of $\frac{\widehat{Q}_\omega}{\rho c \gamma \omega}$ where $\gamma = \sqrt{\frac{k_v}{\omega}}$ is the e-folding decay scale of the anomaly with depth.*

   (b) *Show that the phase of the temperature oscillations at depth lag those at the surface. On what does the lag depend?*

   (c) *For common rock material, $k_v = 10^{-6}\ \text{m}^2\,\text{s}^{-1}$, $\rho = 3000\,\text{kg}\,\text{m}^{-3}$ and $c = 1000\,\text{J}\,\text{kg}^{-1}\,\text{K}^{-1}$. Use your answers in (a) to estimate the vertical scale over which temperature fluctuations decay with depth driven by (i) diurnal and (ii) seasonal variations in $\widehat{Q}_\omega$.*

   If $\widehat{Q}_\omega = 100\,\text{J}\,\text{s}^{-1}\,\text{m}^{-2}$, estimate the magnitude of the temperature fluctuations at the surface over the diurnal and seasonal cycles.

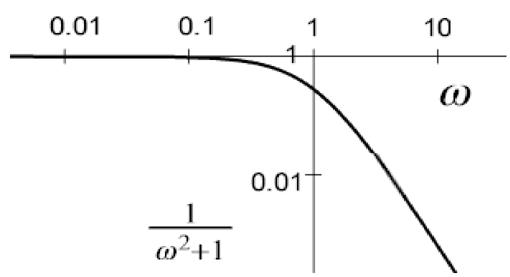
   Comment on your results in view of the fact that the freezing depth — the depth to which soil normally freezes each winter — is about 1 m *in the NE of the US. In areas of central Russia, with extreme winters, the freezing depth can be as much as* 3 m, *compared to, for example, San Francisco where it is only a few centimeters. In the Arctic and Antarctic the freezing depth is so deep that it becomes year-round permafrost. Instead, there is a thaw line during the summer.*

2. *Imagine that the temperature of the ocean mixed layer of depth h, governed by Eq.(12.1), is forced by air-sea fluxes due to weather systems represented by a white-noise process $Q_{\text{net}} = \widehat{Q}_\omega e^{i\omega t}$ where $\widehat{Q}_\omega$ is the amplitude of the forcing at frequency ω. Solve Eq.(12.1) for the temperature response $T = \mathrm{Re}\,\widehat{T}_\omega e^{i\omega t}$ and hence show that:*

$$\widehat{T}_\omega = \frac{\widehat{Q}_\omega}{\gamma_O\left(\frac{\lambda}{\gamma_O} + i\omega\right)}.$$

Hence show that it has a spectrum, $\widehat{T}_\omega \widehat{T}^*_\omega$, where $\widehat{T}^*_\omega$ is the complex conjugate, given by Eq.(12.2). Graph the spectrum using a log-log plot and hence convince yourself that fluctuations with a frequency greater than $\frac{\lambda}{\gamma_O}$ are damped.

Substituting $\widehat{T} = \frac{\widehat{Q}}{\gamma_O\left(i\omega + \frac{\lambda}{\gamma_O}\right)}$ in to Eq.(12.1), we find it is a solution, yielding a spectrum $\widehat{T}\widehat{T}^* = \frac{\widehat{Q}^2}{\gamma_O^2\left(\frac{\lambda}{\gamma_O}+i\omega\right)\left(\frac{\lambda}{\gamma_O}-i\omega\right)} = \frac{\widehat{Q}^2}{\gamma_O^2\left(\omega^2+\left(\frac{\lambda}{\gamma_O}\right)^2\right)}$. The function $\frac{1}{1+\omega^2}$ is plotted in the figure, where ω has been normalized wrt the scale $\frac{\lambda}{\gamma_O}$.



We see that for $\omega^2 >> 1$ (i.e. frequencies greater than $\frac{\lambda}{\gamma_O}$) the spectrum falls off with increasing ω like ω^{-2} and so has a slope of -2 on the log-log plot. These timescales are thus damped. At low frequencies such that $\omega^2 << 1$, the curve asymptotes to unity.

3. *For the one-layer "leaky greenhouse" model considered in Fig.2.8 of Chapter 2, suppose that, all else being fixed, the atmospheric absorption*

depends linearly on atmospheric CO_2 concentration as

$$\epsilon = \epsilon_0 + [\mathrm{CO_2}]\,\epsilon_1\,,$$

where $[CO_2]$ is CO_2 concentration (in ppm), $\epsilon_0 = 0.734$, and $\epsilon_1 = 1.0 \times 10^{-4}(ppm)^{-1}$. Calculate, for this model, the surface temperature:

(a) *for the present atmosphere, with $[CO_2] = 360ppm$ (see Table 1.2);*

(b) *in pre-industrial times, with $[CO_2] = 280ppm$; and*

(c) *in a future atmosphere with $[CO_2]$ doubled from its present value.*

In Section 2.3.2 we deduced for the "leaky greenhouse" model that the surface temperature is related to emission temperature by

$$T_s = \left(\frac{2}{2-\epsilon}\right)^{\frac{1}{4}} T_e\,,$$

where $T_e = \left[(1-\alpha_p)\,\frac{S_0}{4\sigma}\right]^{\frac{1}{4}} = 255\mathrm{K}$ is the emission temperature for the Earth. With

$$\epsilon = \epsilon_0 + [CO_2]\,\epsilon_1$$

and the given numbers, we get the following.

a. Present atmosphere. $[CO_2] = 360\mathrm{ppm} \rightarrow \epsilon = 0.770$. So

$$T_s = \left(\frac{2}{1.230}\right)^{\frac{1}{4}} \times 255 = 288.0\mathrm{K}\,.$$

b. Pre-industrial atmosphere. $[CO_2] = 280\mathrm{ppm} \rightarrow \epsilon = 0.762$. So

$$T_s = \left(\frac{2}{1.238}\right)^{\frac{1}{4}} \times 255 = 287.5\mathrm{K}\,,$$

0.5K cooler than the present.

c. Future atmosphere with doubled CO_2. $[CO_2] = 720\mathrm{ppm} \rightarrow \epsilon = 0.806$. So

$$T_s = \left(\frac{2}{1.194}\right)^{\frac{1}{4}} \times 255 = 290.1\mathrm{K}\,,$$

2.1K warmer than the present.

4. *Faint early sun paradox*

The emission temperature of the Earth at the present time in its history is 255 K. *Way back in the early history of the solar system, the radiative output of the Sun was thought to be* 25% *less than it is now. Assuming all else (Earth-Sun distance, Earth albedo, atmospheric concentration of Greenhouse gases etc) has remained fixed, use the one-layer "leaky greenhouse" model explored in Q.3 to:*

(a) *determine the emission temperature of the Earth at that time if Greenhouse forcing then was the same as it is know. Hence deduce that the earth must have been completely frozen over.*

The emission temperature would have been $(0.75)^{\frac{1}{4}} \times T_{e_{\text{present day}}}$ $= (0.75)^{\frac{1}{4}} \times 255\,\text{K} = 237\,\text{K}$. According to the leaky greenhouse model, then, from our answer to Q.3(a) appropriate to present-day levels of CO_2 forcing, we have $T_s = \left(\frac{2}{2-\epsilon}\right)^{\frac{1}{4}} T_e = \left(\frac{2}{1.230}\right)^{\frac{1}{4}} 237\,\text{K} =$ 268 K. Since this is less than 273 K, the freezing point of water, we conclude that the earth would have been frozen over.

(b) *if the early Earth were not frozen over due to the presence of elevated levels of CO_2, use your answer to Q.3 to estimate how much CO_2 would have had to have been present. Comment on your answer in view of Fig.12.14.*

One requires that $T_s = \left(\frac{2}{2-\epsilon}\right)^{\frac{1}{4}} T_e = 273\,\text{K}$ and so, given that $T_e =$ 237 K, $\left(\frac{2}{2-\epsilon}\right)^{\frac{1}{4}} = \frac{273}{237} = 1.15$ implying that $\epsilon = 0.86$. If the rule used in Q.3 applies:

$$\epsilon = \epsilon_0 + [\text{CO}_2]\,\epsilon_1\,,$$

where $[CO_2]$ is CO_2 concentration (in ppm), $\epsilon_0 = 0.734$, and $\epsilon_1 = 1.0 \times 10^{-4}(\text{ppm})^{-1}$, then a CO_2 concentration of $(0.86 - 0.734) \times 10^4 ppm = (0.86 - 0.734) \times 10^4 = 1260 ppm$ would be required to bring the surface temperature up to the freezing point. This is $\frac{1260}{280} = 4.5$ times pre-industrial values. According to Fig.12.14 levels of CO_2 may be have 20 times present values in the distant past, presumably more than enough to keep the surface of the planet ice-free. In fact the model we have employed here is useful for illustrating underlying principles, but is not appropriate for quantitative study.

5. *Bolide impact*

There is strong evidence that a large meteorite or comet hit the earth about $65M\,\mathrm{y}$ *ago near the Yucatan Peninsula extinguishing perhaps 75% of all life on earth — the K-T extinction marking the end of the Cretaceous (K). It is speculated that the smoke and fine dust generated by the resulting fires would have resulted in intense radiative heating of the mid-troposphere with substantial surface cooling (by as much as* $20\,^{\circ}\mathrm{C}$*) which could interrupt plant photosynthesis and thus destroy much of the Earth's vegetation and animal life.*

A slight generalization of the one-dimensional problems considered in Chapter 2 provide insights in to the problem.

By assuming that a fraction 'f' of the incoming solar radiation in Fig.2.8 is absorbed by a dust layer and that, as before, a fraction 'ϵ' of terrestrial wavelengths emitted from the ground is absorbed in the layer, show that:

$$T_s = \left(\frac{2-f}{2-\epsilon}\right) T_e$$

where T_e *is the given by Eq.(2.4).*

Assume blackbody radiation and that each component of the atmosphere-Earth system is in thermal equilibrium. Consider first the surface heat balance. The net radiative flux per unit area from the surface is σT_s^4, where σ is the Stefan-Boltzmann constant. The surface gains heat from solar radiation, at a rate $\frac{1}{4}S_0\,(1-\alpha_p)\,(1-f)$ per unit area, and from the atmosphere. From Kirchoff's law, the atmosphere radiates both up and down, at a rate $\varepsilon\sigma T_a^4$. So the surface heat balance is

$$\text{net input per unit area} = \text{net loss per unit area}$$

i.e.,

$$\frac{1}{4}S_0\,(1-\alpha_p)\,(1-f) + \varepsilon\sigma T_a^4 = \sigma T_s^4 \ .$$

Using the definition of emission temperature,

$$\frac{1}{4}S_0\,(1-\alpha_p) = \sigma T_e^4 \ ,$$

the surface balance can be written

$$(1-f)\,T_e^4 + \varepsilon T_a^4 = T_s^4 \ . \qquad (1.1)$$

Now, the net loss or gain from space must be zero. The net input per unit area to the Earth is

$$\frac{1}{4}S_0\,(1-\alpha_p)=\sigma T_e^4\ ,$$

while the net loss per unit area is that from the atmosphere, $\varepsilon\sigma T_a^4$, plus that penetrating the atmosphere from the surface, $(1-\varepsilon)\,\sigma T_s^4$. Equating gain with loss (and canceling σ),

$$T_e^4=\varepsilon T_a^4+(1-\varepsilon)\,T_s^4\ . \tag{1.2}$$

Elimination of T_a from (1.1) and (1.2) leaves

$$T_s=\left[\frac{2-f}{2-\varepsilon}\right]^{\frac{1}{4}}T_e\ . \tag{1.3}$$

While atmospheric absorption of IR (represented by ε) increases surface temperature, absorption of shortwave solar radiation (represented by f) decreases T_s, simply by reducing the direct solar input to the surface.

Investigate the extreme case where the dust layer is so black that it has zero albedo (no radiation reflected, $\alpha_p=0$*) and is completely absorbing (*$f=1$*) at solar wavelengths.*

If the atmosphere remains strongly absorbing in the infrared, $\varepsilon\sim 1$, then:

$$T_s\sim\left[\frac{1}{2-1}\right]^{\frac{1}{4}}T_e\sim 280K\text{ if }\alpha_p\ =0$$

a substantial cooling. Perhaps we might expect ε to be reduced somewhat because of the increased stability of the atmosphere (cold surface, hot cloud) which would inhibit convection and hence reduce the water vapor concentration. Putting $\varepsilon=0.8$ gives $T_s=268K$, an even larger drop in temperature. Obviously the more 'transparent' is the dust layer to terrestrial wavelengths, the greater the cooling of the surface.

6. *Assuming that the land ice over the North American continent at the Last Glacial Maximum shown in Fig.12.17 had an average thickness of* 2 km*, estimate the freshwater flux into the adjacent oceans (in Sv) that would have occurred if it had completely melted in* 10 y*,* 100 y*,* 1000 y*.*

Compare your estimates to the observed freshwater meridional flux in the ocean, Fig.11.32. Another useful comparative measure is the flux of the Amazon river, 0.2Sv.

We estimate the surface area occupied by land ice over North America at the LGM to be 2000 km$\times$ 2000 km yielding a volume of 2000 km$\times$ 2000 km $\times$ 2 km $= 8.0 \times 10^{15}$ m^3 if it had been 2 km thick. If all the ice melted in 10 y this yields a flux of $\frac{8.0\times10^{15}\,\mathrm{m}^3}{10\,\mathrm{y}} = 25\times10^7\,\mathrm{m}^3\,\mathrm{s}^{-1} = 250$Sv, a flux of 25Sv if it melted in 100 y, and a flux of 2.5Sv if it melted in 1000 y. These are all much larger than meridional fresh water fluxes in the ocean ($\sim$ 1Sv) and typical river fluxes ($<<$1Sv).