Atmosphere, Ocean and Climate Dynamics
Answers to Chapter 11

1. *It is observed that water sinks in to the deep ocean in polar regions of the Atlantic basin at a rate of* $15Sv$.

   (a) *How long would it take to 'fill up' the Atlantic basin?* $\frac{10^{14}\,\mathrm{m}^2\times 5\times 10^3\,\mathrm{m}}{1.5\times 10^7\,\mathrm{m}^3\,\mathrm{s}^{-1}} = \frac{3.3333\times 10^{10}}{10^5\times 365} = 913.23$

   The area of the Atlantic Ocean is about $10^{14}\,\mathrm{m}^2$. The depth of the ocean is 5 km. So we can estimate of the time scale of the overturning circulation as follows: $\tau = \frac{\text{ocean volume}}{\text{volume flux}} = \frac{10^{14}\,\mathrm{m}^2\times 5\times 10^3\,\mathrm{m}}{1.5\times 10^7\,\mathrm{m}^3\,\mathrm{s}^{-1}} \simeq 900$ y.

   (b) *Supposing that the local sinking is balanced by large-scale upwelling, estimate the strength of this upwelling. Express your answer in* $\mathrm{m\,y^{-1}}$.

   If compensating upwelling occupies almost all of the ocean basin, the upwelling velocity must be about $w = \frac{\text{volume flux}}{\text{area of ocean}} = \frac{1.5\times 10^7\,\mathrm{m}^3\,\mathrm{s}^{-1}}{10^{14}\,\mathrm{m}^2} \simeq 4\,\mathrm{m\,y^{-1}}$(!!), ten times smaller than typical Ekman pumping rates driven by the wind.

   (c) *Assuming that* $\beta v = f\frac{\partial w}{\partial z}$, *infer the sense and deduce the magnitude of the meridional currents in the interior of the abyssal ocean where columns of fluid are being stretched.*

   The meridional velocity, $v = \frac{f}{\beta}\frac{w}{H} \simeq \frac{10^{-4}\,\mathrm{s}^{-1}}{10^{-11}\,\mathrm{s}^{-1}\,\mathrm{m}^{-1}}\frac{4\,\mathrm{m\,y^{-1}}}{3\,\mathrm{km}} = 0.3\times 10^{-3}\,\mathrm{m\,s^{-1}}$, a tiny velocity. This is roughly consistent with an estimate of horizontal flow suggested by continuity which is $v = \frac{\text{volume flux}}{\text{depth}\times\text{width}} = \frac{1.5\times 10^7\,\mathrm{m}^3\,\mathrm{s}^{-1}}{3\times 10^3\,\mathrm{m}\times 5\times 10^6\,\mathrm{m}} \simeq 10^{-3}\,\mathrm{m\,s^{-1}}$.

   The interior horizontal current is directed toward the pole.

   (d) *Estimate the strength of the western boundary current.*

   If the strength of the deep water source is S Sv, the western boundary current carries a total transport of $2\times S$, one unit of S directly from the source and one unit that is recirculating.

2. *Review Section 11.3.3, but now suppose that boundary currents flow northwards in Fig.11.20. By considering the role of boundary current friction in inducing Taylor columns to stretch/compress — Eq.(11.10)*

— *deduce that northward flowing eastern (western) boundary currents are disallowed (allowed).*

Consider a northward flowing eastern (western) boundary current. A paddle wheel placed in it will turn cyclonically (anticyclonically) because flow on its inside flank is faster than on its outside flank. Hence $\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} > 0$ (< 0). Now using Eq.(11.10) we see that $\frac{\partial w}{\partial z} > 0$ (< 0). If the current is flowing northwards (i.e. moving to the shallower end of the tank in GFD Expt XIV) then it must contract $\frac{\partial w}{\partial z} < 0$, which is the wrong (correct) sign if the boundary current is on the east (west). Thus the signs in Eq.(11.10) are inconsistent (consistent) for a northward flowing current of the east (west) of the basin. See the discussion on Section 11.3.3.

3. *Consider the laboratory experiment GFD XV — source sink flow in a rotating basin. Use the Taylor-Proudman theorem and that eastern boundary currents are disallowed, to sketch the pattern of flow taking fluid from source to sink for the scenarios given in Fig.11.33. Note that one of the solutions is given in Fig.11.22!*

   Patterns of flow are sketched in the Fig.1.

4. *From Fig.11.6 one sees that evaporation exceeds precipitation by order $1\,\mathrm{m\,y^{-1}}$ in the subtropics ($\pm 30^o$), but the reverse is true at higher latitudes ($\pm 60^o$).*

   (a) *Estimate the meridional freshwater transport of the ocean required to maintain hydrological balance and compare with Fig.11.32.*

   The area of the ocean between the equator and 30^oN is roughly $\frac{3}{4} \times 10^{14}\,\mathrm{m^2}$. If evaporation exceeds precipitation by $1\,\mathrm{m\,y^{-1}}$ over this band then to enable a steady state to be established, the ocean must transport $\frac{3}{4} \times 10^{14}\,\mathrm{m^2} \times 1\,\mathrm{m\,y^{-1}} \simeq 2 \times 10^6\,\mathrm{m^3\,s^{-1}}$ or 2Sv of fresh water. (The atmosphere transports the same amount of water in the opposite direction). This is of the same order as in Fig.11.32, which is the freshwater water transport in the Atlantic ocean.

   (b) *Latent heat is taken from the ocean to evaporate water which subsequently falls as rain at (predominantly) higher latitudes, as sketched in Fig.11.5. Given that the latent heat of evaporation of water is*

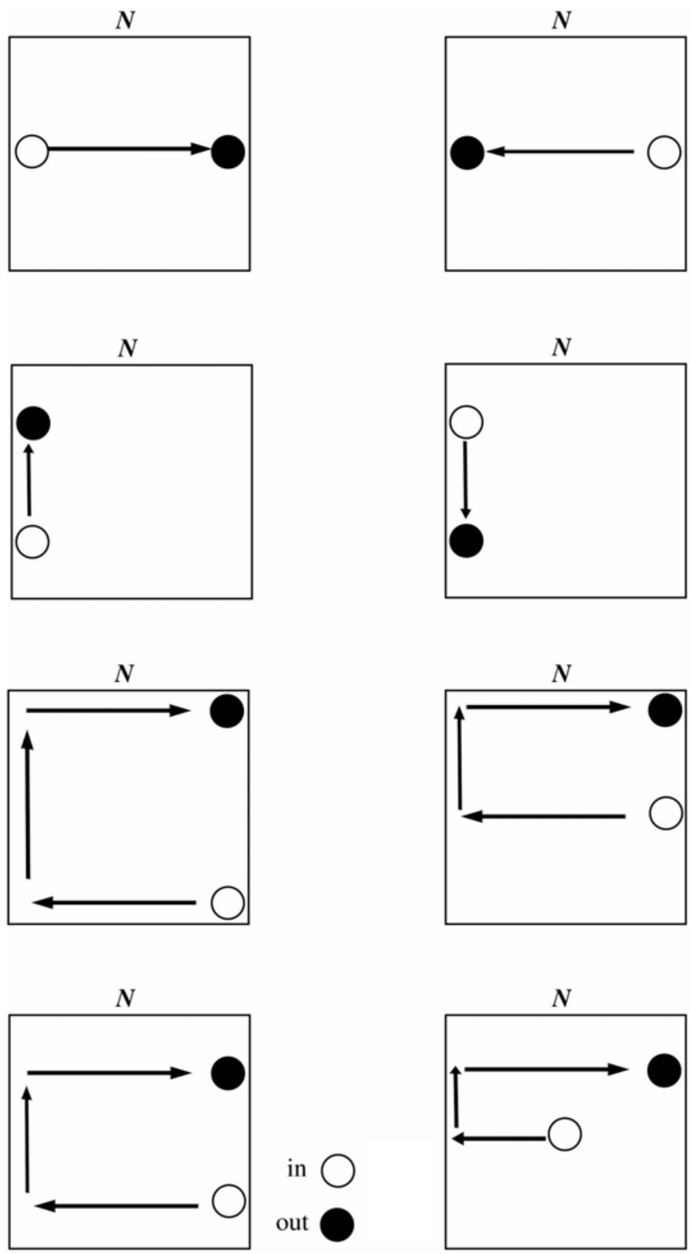

Figure 1: Flow from source to sink in GFD LabXV. Note that Fig.11.22 corresponds to the case at the bottom of the column on the left.

$2.25 \times 10^6\,\mathrm{J\,kg^{-1}}$, *estimate the implied meridional flux of energy in the atmosphere and compare with Fig.8.13.*

The implied meridional atmospheric energy transport is $L \times \rho\times$ (meridional volume transport of water) where L is the latent heat of condensation of water and ρ is the density of water. Inserting numbers we obtain $2.25 \times 10^6\,\mathrm{J\,kg^{-1}} \times 10^3\,\mathrm{kg^{-1}\,m^{-3}} \times 1.5 \times 10^6\,\mathrm{m^3\,s^{-1}} = 3 \times 10^{15}\mathrm{W}$, 3PW, a significant fraction of the meridional energy transport in the atmosphere — see Fig.8.13 and discussion in Section 8.4.1.

5. *In the present climate the volume of freshwater trapped in ice sheets over land is* $\sim 33 \times 10^6\,\mathrm{km}^3$. *If all this ice melted and ran into the ocean, by making use of the data in Table 9.1, estimate by how much the sea level would rise. What would happen to sea level if all the sea-ice melted?*

The surface area of the ocean is $3.61 \times 10^{14}\,\mathrm{m}^2$ and so sea level would rise by $\frac{33\times10^6\,\mathrm{km}^3}{3.61\times10^{14}\,\mathrm{m}^2} \simeq 90\,\mathrm{m}$ if all ice sheets over land melted. Sea level would not be affected by melting of sea-ice because sea-ice is already floating on water displacing its weight.