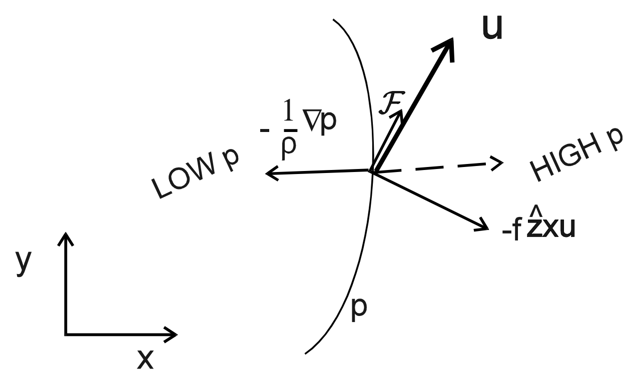
Atmosphere, Ocean and Climate Dynamics
Answers to Chapter 10

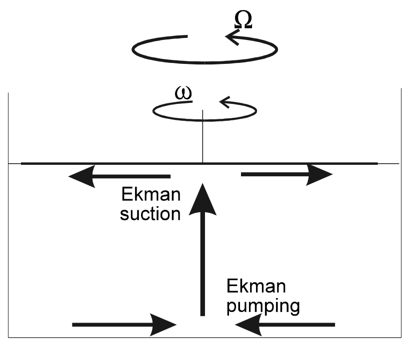
1. *Consider the "Ekman layer" experiment GFD Lab XII, Section 10.1.2. Assume the lid rotates cyclonically with respect to the turntable. In addition to the Ekman layer at the base, there is a second layer at the lid. In this top Ekman layer, the effect of friction is to drive the flow, rather than to slow it down as at the base.*

   (a) *Discuss the nature of the Ekman layer at top: illustrate the balances of forces within it and describe the radial and vertical component of flow within it. Discuss the parallel between the top Ekman layer and the wind-driven boundary layer at the surface of the ocean.*

   Since friction drives the flow in the top Ekman layer, the friction force $\mathcal{F}$ will be approximately parallel (rather than antiparallel) to the flow. For cyclonic rotation, the balance must be as shown below:



   The flow thus spirals out of the cyclone at the surface, rather than into it as in a bottom boundary layer. Thus, the radial component of the flow in the tank must look qualitatively as follows:



— Ekman pumping at the bottom, producing upwelling, which is absorbed by the Ekman suction at the lid.

(b) *Discuss the parallel between the bottom Ekman layer and the atmospheric boundary layer.*

At the bottom the friction force $\mathcal{F}$ will be approximately antiparallel to the flow (because it opposes, rather than drives the flow), as in Fig.7.24 of our notes. This is just as observed in the atmospheric boundary layer — see Fig.7.25.

2. *Fig.10.11 shows the pattern and magnitude of Ekman pumping acting on the ocean. Estimate how long it would take a particle of fluid to move a vertical distance of* $1\,\mathrm{km}$ *if it had a speed* w_{Ek}. *If properties are diffused vertically at a rate* $k = 10^{-5}\,\mathrm{m^2\,s^{-1}}$ *(typical of the main thermocline), compare this to the implied diffusive time-scale. Comment.*

   Typical Ekman pumping vertical velocities are $30\,\mathrm{m\,y^{-1}}$. A fluid parcel moving at this speed would traverse a distance of $1\,\mathrm{km}$ in a time $t_{\mathrm{advective}} = \frac{1\,\mathrm{km}}{30\,\mathrm{m\,y^{-1}}} \simeq 10^9\,\mathrm{s} \simeq 30\,\mathrm{y}$. The time it takes to diffuse a signal a distance h is $t_{\mathrm{diffusive}} = \frac{h^2}{\pi k}$ where k is the diffusivity. Inserting numbers we obtain a time $t_{\mathrm{diffusive}} = \frac{1\,\mathrm{km}^2}{\pi \times 10^{-5}\,\mathrm{m^2\,s^{-1}}} \simeq 3 \times 10^{10}\,\mathrm{s} = 800\,\mathrm{y}$. We conclude that in the main thermocline advective processes dominate over diffusive processes.

3. *Use the results of Ekman theory to show that when one adds the meridional volume transport in the Ekman layer — given by Eq.(10.5) — to the meridional transport in the geostrophic interior — obtained from*

Eq.(10.12) — one obtains the Sverdrup transport, Eq.(10.17):

$$\frac{1}{\rho_{ref}}\mathbf{M}_{Ek_y} + \int_{-D}^{-\delta} v_g \; dz = \text{Sverdrup transport.}$$

The meridional component of $\mathbf{M}_{Ek} = \frac{\boldsymbol{\tau}_{wind}\times\widehat{\mathbf{z}}}{f}$ is $-\frac{\tau_{wind_x}}{f}$.
Evaluating the vertical integral, we obtain:

$$\begin{aligned}
\int_{-D}^{-\delta} v_g dz &= \frac{f}{\beta}\int_{-D}^{-\delta}\frac{\partial w}{\partial z}dz = \frac{f}{\beta}\left(w(-\delta) - w(-D)\right) = \frac{f}{\beta}w(-\delta) \\
&= \frac{f}{\beta}\frac{1}{\rho_{ref}}\widehat{\mathbf{z}}\cdot\nabla\times\left(\frac{\boldsymbol{\tau}_{wind}}{f}\right) = \frac{f}{\beta}\frac{1}{\rho_{ref}}\left(\frac{\partial}{\partial x}\frac{\tau_{wind_y}}{f} - \frac{\partial}{\partial y}\frac{\tau_{wind_x}}{f}\right)
\end{aligned}$$

Adding the two together we obtain:

$$\begin{aligned}
-\frac{\tau_{wind_x}}{f} + \frac{f}{\beta}\frac{1}{\rho_{ref}}\left(\frac{\partial}{\partial x}\frac{\tau_{wind_y}}{f} - \frac{\partial}{\partial y}\frac{\tau_{wind_x}}{f}\right) &= \frac{1}{\beta}\frac{1}{\rho_{ref}}\left(\frac{\partial\tau_{wind_y}}{\partial x} - \frac{\partial\tau_{wind_x}}{\partial y}\right) \\
&= \frac{1}{\beta}\frac{1}{\rho_{ref}}\widehat{\mathbf{z}}\cdot\nabla\times\boldsymbol{\tau}_{wind} \\
&= \text{Sverdrup transport.}
\end{aligned}$$

4. C*onsider the Atlantic Ocean to be a rectangular basin, centered on* $35^o N$*, of longitudinal width* $L_x = 5000 km$ *and latitudinal width* $L_y = 2500 km$*. The ocean is subjected to a zonal wind stress of the form*

$$\begin{aligned}
\tau_x(y) &= -\tau_0\cos\left(\pi\frac{y}{L_y}\right) \; ; \\
\tau_y(y) &= 0 \; ;
\end{aligned}$$

where $\tau_0 = 0.05 N m^{-2}$*. Assume a constant value of* $\beta = df/dy$ *appropriate to* $35^o N$*, and that the ocean has uniform density* $1000 kg\ m^{-3}$*.*

   (a) *Determine the magnitude and spatial distribution of the depth-integrated northward flow velocity in the interior of the ocean.*

(b) *Using the depth-integrated continuity equation, and assuming no flow at the eastern boundary of the ocean, determine the magnitude and spatial distribution of the depth-integrated eastward flow in the interior.*

(c) *If the return flow at the western boundary is confined to a width of 100km, determine the depth-integrated flow in this boundary current.*

(d) *If the flow is confined to the top 500m of the ocean (and is uniform with depth in this layer), determine the northward components of flow velocity in the interior, and in the western boundary current.*

(a) Assuming a flat ocean bottom with no stress there, then the Sverdrup relation applies (Eq.10.13 of our notes), viz.:

$$V = \frac{1}{\beta \rho_0} \left(\frac{\partial \tau_y}{\partial x} - \frac{\partial \tau_x}{\partial y} \right) ,$$

where V is the depth-integrated northward flow. Substituting, with $\tau_y = 0$,

$$\begin{aligned} V &= -\frac{1}{\beta \rho_0} \frac{\pi}{L_y} \tau_0 \sin\left(\pi \frac{y}{L_y} \right) \\ &= V_0 \sin\left(\pi \frac{y}{L_y} \right) , \end{aligned}$$

where, with $\beta(35^o) = \frac{2\Omega}{a} \cos 35^o = 1.87 \times 10^{-11} \mathrm{m}^{-1}\mathrm{s}^{-1}$,

$$V_0 = -\frac{\pi \times 0.05}{1.87 \times 10^{-11} \times 1000 \times 2.5 \times 10^6} = -3.4 \mathrm{m}^2\mathrm{s}^{-1}.$$

(b) Depth-integrated continuity equation is

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0 ,$$

where U is the depth-integrated eastward flow. Since $\partial U / \partial x$ is independent of x, and U vanishes at the eastern boundary $x = L_x$, we can write this as

$$U = U_1(y) \left[1 - \frac{x}{L_x} \right] ,$$

when continuity gives us

$$-\frac{U_1}{L_x}+\pi\frac{V}{L_y}\cos\left(\pi\frac{y}{L_y}\right)=0$$

whence

$$U=U_1(y)\left[1-\frac{x}{L_x}\right]=U_0\left[1-\frac{x}{L_x}\right]\cos\left(\pi\frac{y}{L_y}\right)$$

where

$$U_0=\pi\frac{L_x}{L_y}V_0=-3.4\times\pi\times 2=-21.4\ \mathrm{m^2s^{-1}}.$$

(c) The flow described by these answers is valid only in the ocean interior. A return flow must exist at the western boundary. The longitudinally and depth-integrated northward flow (net volume flux) in the ocean interior is

$$\mathcal{V}=\int_0^{L_x}V\ dx=V_0L_x\sin\left(\pi\frac{y}{L_y}\right).$$

If the return boundary flow has velocity V_b, uniform over a width $L_b=100$km, then, since the volume fluxes must be the same,

$$V_bL_b=-\mathcal{V}=-V_0L_x\sin\left(\pi\frac{y}{L_y}\right)\ ,$$

whence

$$V_b=V_{b0}\sin\left(\pi\frac{y}{L_y}\right)$$

where

$$V_{b0}=-V_0\frac{L_x}{L_b}=170\mathrm{m^2s^{-1}}.$$

(d) If the flow is confined to the top 500m, the northward components of the flow are

$$v=\frac{V_0}{500\mathrm{m}}\sin\left(\pi\frac{y}{L_y}\right)=-\left(6.8\times10^{-3}\mathrm{m\ s^{-1}}\right)\sin\left(\pi\frac{y}{L_y}\right)\ ,$$

and

$$v=\frac{V_b}{500\mathrm{m}}=\left(0.34\mathrm{ms^{-1}}\right)\sin\left(\pi\frac{y}{L_y}\right)$$

in the western boundary current.

5. *From your answer to Question 4, determine the net volume flux at $35^o N$ (the volume of water crossing this latitude per unit time)*

   (a) *for the entire ocean excluding the western boundary current*

   (b) *for the western boundary current only. Give your answers in units of Sverdrups (Sv) [$1 Sv = 10^6 m^3 s^{-1}$; the Sverdrup is a conventional unit of water flow in oceanography.*

   (c) *Assume again that the flow is confined to the top $500m$ of the ocean. Determine the volume of the top $500m$ of the ocean and, by dividing this number by the volume flux you calculated in part (a), come up with a time scale. Discuss what this time scale means.*

   (d) *Given that the circulation is confined to the top* 500m, *assume now that the water in the western boundary current has a mean temperature of $20^o C$, while the rest of the ocean has a mean temperature of* 15^oC. *Show that H, the net flux of heat across $35^o N$, is*

   $$H = \rho_0 c_p \mathcal{V}\, \Delta T \ ,$$

   where $\mathcal{V}$ is the volume flux you calculated in question 3 and ΔT is the temperature difference between water in the ocean interior and in the western boundary current. [Density of water $\rho_0 = 1000 kg\ m^{-3}$; specific heat of water $c_p = 4187 J\ kg^{-1} K^{-1}$.] We earlier found that the Earth's energy balance requires a poleward heat flux of around $5 \times 10^{15}\,W$. Calculate and discuss what contribution the Atlantic Ocean makes to this flux.

(a) In the channel center at 35^oN, $y = L_y/2$, and $\sin(\pi y/L_y) = 1$. In the ocean interior, the net volume flux is, from the answer to 2(c),

$$\begin{aligned} \mathcal{V} &= V_0 L_x \sin\left(\pi \frac{y}{L_y}\right) , \\ &= V_0 L_x \quad \text{at } 35^o\text{N} . \end{aligned}$$

Therefore the interior volume flow at 35^oN is $\mathcal{V}(35^o\text{N}) = V_0 L_x = -1.7 \times 10^7 \text{m}^3\text{s}^{-1} = -17\text{Sv}$ (southward).

(a) (b) The volume flow in the western boundary current must be the same—that is how I determined the strength of the boundary current. A a check, the volume flow in the boundary is

$$
\begin{aligned}
V_b L_b &= \mathcal{V}_b = V_{b0} L_b \sin\left(\pi \frac{y}{L_y}\right) \\
&= V_{b0} L_b \text{ at } 35^o\text{N}.
\end{aligned}
$$

This has the value $170\text{m}^2\text{s}^{-1} \times 100\text{km} = 1.7 \times 10^7\text{m}^3\text{s}^{-1} = 17\text{Sv}$, directed northward.

(c) The ocean in Q2, has surface area $2500\text{km}\times 5000\text{km} = 1.25 \times 10^{13}\text{m}^2$, and so the volume of the upper 500m of the ocean i $6.25 \times 10^{15}\text{m}^3$. Therefore we can define a time scale τ where

$$
\begin{aligned}
\tau &= \frac{\text{volume}}{\text{volume flux}} \\
&= \frac{6.25 \times 10^{15}\text{m}^3}{1.7 \times 10^7\text{m}^3\text{s}^{-1}} \\
&= 3.68 \times 10^8\text{s} \\
&= 11.7 \text{ years.}
\end{aligned}
$$

This is a gross measure of the time it takes for the volume flux $\mathcal{V}$ to replenish the upper ocean.

(d) The interior volume flux across 35^oN is $\mathcal{V}$, calculated above. Thus, in the interior, there is an amount $\mathcal{V}$ going south in the interior, and an equal amount going north in the western boundary. Neglecting variations in water density, there is therefore a mass flux $\rho_0\mathcal{V}$ going south in the interior—and so $(\rho_0\mathcal{V})$ kg of water crosses 35^oN every second—and an equal amount going north in the western boundary current (so there is, of course, no net mass transport). That going south has temperature T_i, and heat content per unit mass c_pT_i, and so the amount of heat crossing 35^oN per unit time, southward, in the interior is $c_p\rho_0\mathcal{V}T_i$. Similarly, if the temperature at the western boundary is T_b, the northward transport of heat in the boundary current is $c_p\rho_0\mathcal{V}T_b$. Hence the net northward heat transport is

$$
\begin{aligned}
H &= \text{(northward flux in boundary current)} \\
&\quad - \text{(southward flux in interior)} \\
&= c_p\rho_0\mathcal{V}\,\Delta T
\end{aligned}
$$

where $\Delta T = T_b - T_i$ is the temperature difference between the boundary current and the open ocean.

Using the numbers from preceding questions, and $\Delta T = 5^o$,

$$\begin{aligned} H &= 4187 \times 1000 \times 1.7 \times 10^7 \times 5 \\ &= 0.36 \times 10^{15}\mathrm{W}. \end{aligned}$$

This is a small but significant fraction (about 7%) of the required net poleward flux (from both ocean and atmosphere) of 5×10^{15}W, suggesting that the wind-driven circulation of the Atlantic Ocean plays a role. [Actually, in total the N Atlantic contributes about 1×10^{15}W, and a substantial part of this is due to the thermohaline circulation.]

6. *Describe how the design of the laboratory experiment sketched in Fig.10.18 captures the essential mechanism behind the wind-driven ocean circulation. By comparing Eq.(10.16) with Eq.(10.12), show that the slope of the bottom of the laboratory tank plays the role of the $\beta-$effect: i.e. bottom slope $\longleftrightarrow \frac{1}{\tan\varphi}\frac{h}{a}$ where h is the depth of the ocean and a is the radius of the earth.*

   Writing Eq.(10.12) thus: $\frac{h\beta}{f}v = w_{Ek}$ and comparing with Eq.(10.16), and noting the definitions of f and β (Eqs.6.42 and 10.10), one sees that $\frac{\partial d}{\partial y} \longleftrightarrow \frac{1}{\tan\varphi}\frac{h}{a}$. Hence one can use a sloping false bottom in the laboratory to mimic the effect of the variation of the Coriolis parameter with latitude. The rotating disc at the surface represents the action of the wind blowing over the surface of the water.

7. *Imagine that the Earth was spinning in the opposite direction to the present.*

   (a) *What would you expect the pattern of surface winds to look like, and why (read again Chapter 8)?*

   If the earth were spinning in the opposite direction, winds in the northern hemisphere would have the sense of direction that an observer in the southern hemisphere on the present earth would detect if they stood facing toward the south pole. The winds would therefore blow predominantly from the observer's right to left. We see then that the sense of direction of the winds would have reversed.

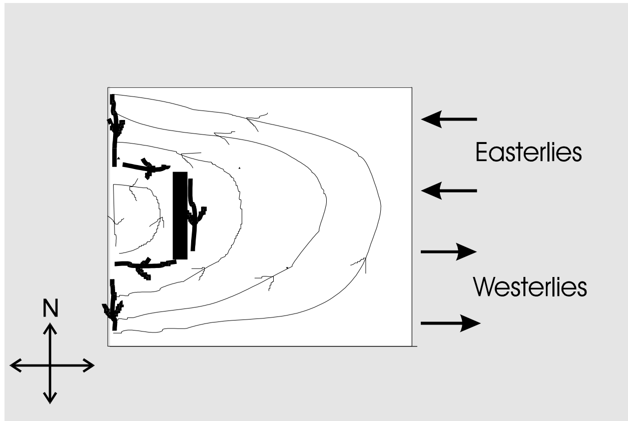


Figure 1:

(b) *on what side (east or west) of the ocean basins would you expect to find boundary currents in the ocean, and why?*

Boundary currents would occur on the eastern margin of the ocean basins because the sign of β, the meridional gradient of the planetary vorticity, would be of opposite sign.

If you live in the southern hemisphere perhaps you are not scratching your head.

8. *Use Sverdrup theory and the idea that only western boundary currents are allowed, to sketch the pattern of ocean currents you would expect to observe in the basin sketched below in which there is an island. Assume a wind pattern of the form sketched in the diagram.*

   The pattern is sketched in the diagram.

9. *Fig.5.5 shows the observed net radiation at the top of the atmosphere as a function of latitude. Taking this as a starting point, describe the chain of dynamical processes that leads to the existence of anticyclonic gyres in the upper subtropical oceans. Be sure to discuss the key physical mechanisms and constraints involved in each step.*

   One has to discuss (i) the pole-equator temperature gradient and associated thermal wind balance (Section 8.2.2) (ii) the baroclinic instability of the thermal wind generating middle-latitude synoptic scale systems (iii) the role of synoptic eddies in meridional angular momentum transport, maintaining middle latitude surface westerlies and tropical easterlies (see Section 8.4.2) (iv) the resulting pattern of Ekman pumping over the ocean (see Section 10.1.3) (v) the Sverdrup response of the ocean (see Section 10.2 and 10.3).