Atmosphere, Ocean and Climate Dynamics
Answers to Chapter 6

1. *Consider the typical, zonally averaged flow, u, shown in Fig.5.20. Concentrate on the vicinity of the subtropical jet near 30°N in winter (DJF). If the x-component of the frictional force per unit mass is*

$$\mathcal{F}_x = \nu \nabla^2 u \ ,$$

where the kinematic viscosity coefficient is $\nu = 1.34 \times 10^{-5} m^2 s^{-1}$ and $\nabla^2 \equiv \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$. Compare the magnitude of this eastward force with the northward or southward Coriolis force and thus convince yourself (and me!) that the frictional force is negligible. [10^o of latitude $\simeq$ $1100 km$; the jet is at an altitude of about $10 km$. You should find that an order-of-magnitude calculation will suffice to make the point unambiguously.]

Frictional force is

$$\mathcal{F}_x = \nu \nabla^2 u \sim \nu \frac{U}{H^2}$$

where H is the scale over which the zonal wind of speed U varies.

The Coriolis force is

$$C \sim fU$$

Thus the ratio of the two terms is:

$$\frac{frictional}{Coriolis} \sim \frac{\nu \frac{U}{H^2}}{fU} = \frac{\nu}{H^2 f} = 10^{-9}$$

if $H = 10 km$. Thus the frictional term is utterly negligible compared to Coriolis.

2. *Using only the equation of hydrostatic balance and the rotating equation of motion, show that a fluid cannot be motionless unless its density is horizontally uniform. (Do not assume geostrophic balance, but you should assume that a motionless fluid is subjected to no frictional forces.)*

The eq. of motion in a rotating frame is

$$\frac{D\mathbf{u}}{Dt} + f\widehat{\mathbf{z}} \times \mathbf{u} = -\frac{1}{\rho}\nabla p - g\widehat{\mathbf{z}} + \mathcal{F} \ .$$

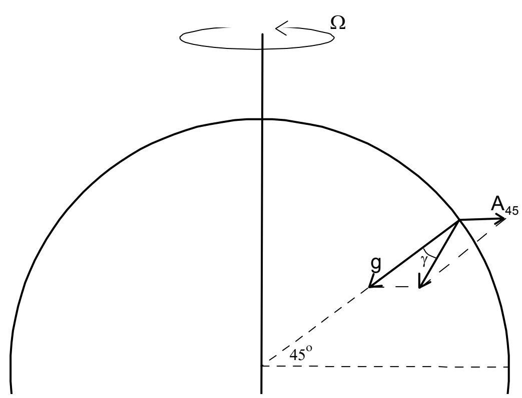
If friction vanishes in a fluid that is everywhere motionless then, for $\mathbf{u} = 0$, the horizontal gradients of pressure must be zero everywhere, since it would otherwise be the only nonzero term in the horizontal components of the eq. of motion. But, from hydrostatic balance

$$\frac{\partial p}{\partial z} = -g\rho \ ,$$

$$\frac{\partial}{\partial z}\nabla p = -g\nabla \rho \ .$$

So, if the horizontal gradient of pressure is everywhere zero, the horizontal gradient of density must also be zero. So a motionless fluid requires no horizontal gradients of density; conversely, a fluid cannot be motionless unless density is horizontally uniform.

3. *a. What is the value of the centrifugal acceleration of a particle fixed to the earth at the equator and how does it compare to g? What is the deviation of a plumb line from the true direction to the centre of the earth at $45°N$?*



The centrifugal acceleration at the equator is just $\Omega^2 a$, where $\Omega = 2\pi/86400 = 7.27 \times 10^{-5}\mathrm{s}^{-1}$ is the Earth's rotation rate and a is the Earth radius, which we here take to be 6370 km. Then the centrifugal acceleration at the equator is just $A_{eq} = \Omega^2 a = 0.034\ \mathrm{ms}^{-2}$, a factor 3.5×10^{-3} smaller than gravity. At 45^oN, the centrifugal acceleration is $A_{45} = \Omega^2 r_{45}$, where $r_{45} = a\cos(45^0)$ is the distance from the surface to the rotation axis. Then $A_{45} = 0.024\ \mathrm{m\,s^{-2}}$, directed outward, normal to the rotation axis, as illustrated in the figure. This acceleration has a vertical component (normal to the surface) of $A_{45}\cos(45^o) = 0.017\mathrm{ms}^{-2}$ and a horizontal component (directed equatorward) of $A_{45}\sin(45^o) = 0.017\ \mathrm{ms}^{-2}$. Hence the net (gravity + centrifugal) acceleration is directed at an angle γ to the downward vertical, where

$$\tan\gamma = \frac{A_{45}\sin(45^o)}{g - A_{45}\cos(45^o)} = 0.0017\ .$$

Since this number is small, we may use the small angle approximation, so that $\gamma = 0.0017$ rad $= 0.097^o$.

b. By considering the centrifugal acceleration on a particle fixed to the surface of the earth, obtain an order-of-magnitude estimate for the earth's ellipticity $\left[\frac{r_1 - r_0}{r_0}\right]$*, where* r_1 *is the equatorial radius and* r_0 *is the polar radius. As a simplifying assumption the gravitational contribution to* g *may be taken as constant and directed toward the centre of the earth. Discuss your estimate given that the ellipticity is observed to be* $\frac{1}{297}$*. You may assume that the mean radius of the earth is* 6000 km.

The total potential Φ (incorporating both gravity and centrifugal effects) for a spherical earth is given by Eq.(6.30):

$$\Phi = gz - \frac{1}{2}\Omega^2 r^2,$$

where r is the distance from the rotation axis and z the height above the spherical reference surface of radius r_{ref}. Note that we have here assumed that gravity is directed toward the earth's center, with a constant magnitude g. Now, we can estimate the shape of the earth by assuming the actual surface to be one of uniform potential. Assuming variations of surface height to be small, $r \simeq r_{ref}\cos\varphi$, where φ is latitude. Then the surface is defined by

$$gz(\varphi) = C + \frac{1}{2}\Omega^2 r_{ref}^2 \cos^2\varphi\ ,$$

where C is a constant. Hence the height difference between the equator ($\cos\varphi = 1$) and poles ($\cos\varphi = 0$) is, using the answer to part (a),

$$r_1 - r_0 = \frac{1}{2g}\Omega^2 r_{ref}^2 = \frac{(7.27\times 10^{-5}\times 6\times 10^6)^2}{2\times 9.81} = 9698\ \mathrm{m}$$

and so our estimate of the earth's ellipticity is $(r_1 - r_0)/r_0 \simeq (r_1 - r_0)/r_{ref} \simeq 9698/(6 \times 10^6) \simeq 1.6 \times 10^{-3}$. This is less than one half of the observed value of $1/297 = 3.4 \times 10^{-3}$. [We have left out a number of factors, the most important being gravity feedback: the distortion of the gravity field associated with the departure of the earth's shape from a perfect sphere is a significant factor, and intensifies the equatorial bulge beyond what our simplified calculation predicts.]

4. *A punter kicks a football a distance of $60m$ on a field at latitude $45°N$. Assuming the ball, until being caught, moves with a constant forward velocity (horizontal component) of $15ms^{-1}$, determine the lateral deflection of the ball from a straight line due to the Coriolis effect. [Neglect friction and any wind or other aerodynamic effects.]*

   Neglecting all other forces, then if u is the velocity component in the direction it is kicked, and v the component normal to this (to the right of u), then, neglecting other forces,

$$\frac{\partial v}{\partial t} = fu$$

   where f is the Coriolis parameter at 45°N. Given that $u = 15\text{ms}^{-1}$ is constant,

$$v = fut \ ,$$

   and so the displacement, to the right of the kicking direction is

$$y = \frac{1}{2} fut^2 \ .$$

   Since the travel time $t = L/u$, where L is the distance traveled,

$$y = \frac{1}{2}\frac{fL^2}{u} \ .$$

   With the given numbers,

$$y = \frac{1.03 \times 10^{-4} \times 60^2}{2 \times 15} = 0.012\text{m} \ = 1.2\text{cm}.$$

   NOTE: (1)The ratio of the lateral displacement to the distance traveled is just

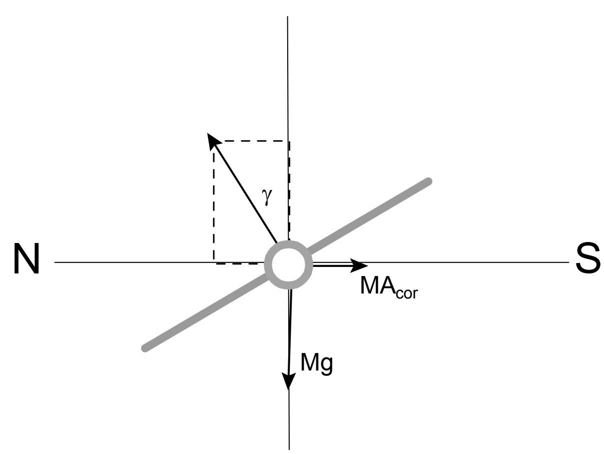
$$\frac{y}{L} = \frac{1}{2}\frac{fL}{u} \ ,$$

i.e., just one half of the inverse of the Rossby number based on the kicking velocity and distance traveled. (2) We have here neglected the action of Coriolis forces of the vertical component of the motion. This is not really valid—the scaling argument we applied to a shallow atmosphere does not apply to the ball's trajectory—but, in fact, the net effect is zero.

5. *Imagine that Concord is (was) flying at speed u from New York to London along a latitude circle. The deflecting force due to Coriolis is toward the south. By lowering the left wing ever so slightly the pilot (or perhaps more conveniently the computer on board) can balance this deflection. Draw a diagram of the forces — gravity, uplift normal to the wings and Coriolis — and use it to deduce that the angle of tilt, γ, of the aircraft from the horizontal required to balance the Coriolis force is*

$$\tan\gamma = \frac{2\Omega\sin\varphi \times u}{g},$$

where Ω is the Earth's rotation, the latitude is φ and gravity is g. If $u = 600\,\mathrm{m\,s^{-1}}$, insert typical numbers to compute the angle. What analogies can you draw with atmospheric circulation? [Hint: cf Eq.(7.8).]



Consider the figure. The southward Coriolis acceleration (at latitude $\varphi = 45^o$N), with $u = 600$ ms^{-1}, is just $A_{cor} = 2\Omega\sin\varphi \times u = 0.062$ ms^{-2}, much less than gravity. The vertical lift force acting on the aircraft required to balance gravity is just Mg, where M is the aircraft's mass. Tilting the aircraft at an angle γ produces a northward horizontal component of this force equal to $Mg\tan\gamma$, which is required to balance the Coriolis force MA_{cor}. Hence $g \tan\gamma = A_{cor}$, or

$$\tan\gamma = \frac{2\Omega \sin\varphi \times u}{g} .$$

With the given numbers, $\tan\gamma = 0.062/9.81 = 6.3 \times 10^{-3}$, or (since this angle is small) $\gamma = 6.3 \times 10^{-3}$ rad $= 0.36^o$. Note that this equation for the required slope directly parallels that of pressure surfaces in geostrophic balance with the zonal wind u: from Eq.(7.8), we have

$$\frac{\partial z}{\partial y} = -\frac{2\Omega \sin\varphi}{g} u ,$$

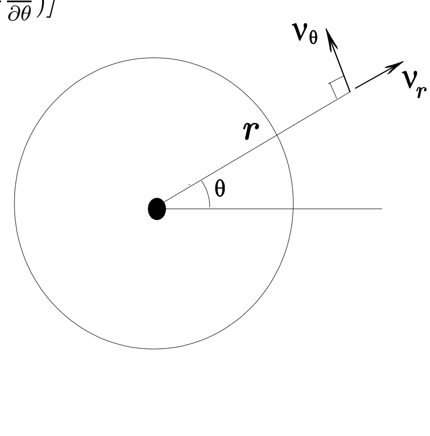
and the meridional slope of the pressure surfaces, γ_p, is such that $\tan\gamma_p = -\partial z/\partial y$.

6. *Consider horizontal flow in circular geometry in a system rotating about a vertical axis with a steady angular velocity* Ω.

   Starting from Eq.6.29 of our notes, show that the equation of motion for the azimuthal flow in this geometry is, in the rotating frame (neglecting friction and assuming 2-dimensional flow)

$$\frac{Dv_\theta}{Dt} + 2\Omega v_r \equiv \frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_\theta v_r}{r} + 2\Omega v_r = -\frac{1}{\rho r}\frac{\partial p}{\partial \theta} \qquad (1)$$

   where (v_r, v_θ) *are the components of velocity in the* (r, θ) = *(radial, azimuthal) directions (see figure). [Hint — write out Eq.6.29 in cylindrical coordinates, noting that* $v_r = \frac{Dr}{Dt}$; $v_\theta = r\frac{D\theta}{Dt}$ *and that the gradient operator is* $\nabla = (\frac{\partial}{\partial r}, \frac{1}{r}\frac{\partial}{\partial \theta})$*]*



The horizontal component of Eq.6.29 is:

$$\frac{D}{Dt}_h \mathbf{u}_h + \frac{1}{\rho}\nabla_h p = -\left(2\boldsymbol{\Omega}\times\mathbf{u}\right)_h .$$

The azimuthal component of the above is:

$$\frac{\partial v_\theta}{\partial t} + \left[\mathbf{u}_h\cdot\nabla\mathbf{u}_h\right]_{\widehat{\theta}} + \frac{1}{r\rho}\frac{\partial}{\partial\theta}p = -2\left[\Omega\times\mathbf{u}_h\right]_{\widehat{\theta}}$$

where $\mathbf{u}_h = (v_r, v_\theta)$. Noting that $\nabla = (\frac{\partial}{\partial r}, \frac{1}{r}\frac{\partial}{\partial\theta})$ in cylindrical coordinates, and being careful to note that ∇ may operate on unit vectors as well, we have:

$$\left[\mathbf{u}_h\cdot\nabla\mathbf{u}_h\right]_{\widehat{\theta}} = \left[\left(v_r\frac{\partial}{\partial r}, \frac{v_\theta}{r}\frac{\partial}{\partial\theta}\right)\mathbf{u}_h\right]_{\widehat{\theta}}$$

Now $\mathbf{u}_h = v_r\widehat{r} + v_\theta\widehat{\theta}$ and so:

$$\left[(v_r\frac{\partial}{\partial r}, \frac{v_\theta}{r}\frac{\partial}{\partial\theta})\mathbf{u}_h\right]_{\widehat{\theta}} = \left[\left(v_r\frac{\partial}{\partial r}, \frac{v_\theta}{r}\frac{\partial}{\partial\theta}\right)\left(v_r\widehat{r} + v_\theta\widehat{\theta}\right)\right]_{\widehat{\theta}}$$

Noting that $\frac{\partial\widehat{r}}{\partial\theta} = \widehat{\theta}$; $\frac{\partial\widehat{\theta}}{\partial\theta} = -\widehat{r}$;$\frac{\partial\widehat{r}}{\partial r} = 0$;$\frac{\partial\widehat{\theta}}{\partial r} = 0$ we get:

$$\begin{aligned}\frac{D}{Dt}_h v_\theta &= \frac{\partial v_\theta}{\partial t} + \underbrace{v_r\frac{\partial v_r}{\partial r}\widehat{r}}_{\text{not in }\widehat{\theta}\text{ direction}} + \underbrace{v_r^2\frac{\partial\widehat{r}}{\partial r}}_{\frac{\partial\widehat{r}}{\partial r}=0} + \underbrace{\frac{v_\theta}{r}\frac{\partial v_r}{\partial\theta}\widehat{r}}_{\text{not in }\widehat{\theta}\text{ direction}} + \frac{v_\theta v_r}{r}\frac{\partial\widehat{r}}{\partial\theta} + v_r\frac{\partial v_\theta}{\partial r}\widehat{\theta} + \\ &\quad \underbrace{v_\theta v_r\frac{\partial\widehat{\theta}}{\partial r}}_{\frac{\partial\widehat{\theta}}{\partial r}=0} + \underbrace{\frac{v_\theta^2}{r}\frac{\partial\widehat{\theta}}{\partial r}}_{\text{not in }\widehat{\theta}\text{ direction}} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial\theta}\widehat{\theta}\end{aligned}$$

Thus:

$$\frac{D}{Dt}_h v_\theta = \frac{\partial v_\theta}{\partial t} + \frac{v_\theta v_r}{r}\frac{\partial\widehat{r}}{\partial\theta} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial\theta}$$

Since $\left[(2\boldsymbol{\Omega}\times\mathbf{u})_h\right]_{\widehat{\theta}} = -2\Omega v_r$, we arrive at the answer! A pretty involved calculation, the complications arising from the fact that ∇ acts on the unit vector.

(a) *Assume that the flow is axisymmetric (i.e., all variables are independent of θ). For such flow, angular momentum (relative to an inertial frame) is conserved. This means, since the angular momentum per unit mass is*

$$m = \Omega r^2 + v_\theta r \ , \tag{2}$$

that

$$\frac{Dm}{Dt} \equiv \frac{\partial m}{\partial t} + v_r \frac{\partial m}{\partial r} = 0 \ . \tag{3}$$

Show that Eqs.(1) and (3) are mutually consistent for axisymmetric flow.

If the flow is axisymmetric, then:

$$\frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_\theta v_r}{r} + 2\Omega v_r = -\frac{1}{\rho r}\frac{\partial p}{\partial \theta}$$

reduces to:

$$\frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta v_r}{r} + 2\Omega v_r = 0$$

Expanding out Eq.(3), $\frac{\partial m}{\partial t} + v_r \frac{\partial m}{\partial r} = 0$, using the definition of m, Eq.(2), we arrive at the above.

(b) *When water flows down the drain from a basin or a bath tub, it usually forms a vortex. It is often said that this vortex is anticlockwise in the northern hemisphere, and clockwise in the southern hemisphere. Test this saying by doing the following.*

Fill a basin or a bath tub (preferably the latter—the bigger the better) to a depth of at least 10cm, let it stand for a minute or two, and then let it drain. When a vortex forms[1]*, estimate, as well as you can, its angular velocity, direction, and radius (floating some small floats, such as pencil shavings, will help to see the flow). Hence calculate the angular momentum per unit mass of the vortex.*

Now, suppose that, at the instant you opened the drain, there was

[1] A clear vortex (with a "hollow" center) may not form. As long as there is an identifiable swirling motion, you will be able to proceed; if not, try repeating the experiment.

no motion (relative to the rotating Earth). Now if only the vertical component of the Earth's rotation matters, calculate the angular momentum density due to the Earth's rotation at the perimeter of the bath tub or basin. [Your tub or basin will almost certainly not be circular, but assume it is, with an effective radius R such that the area of your tub or basin is πR^2 in order to determine m.]

Suppose, in a bathtub of dimension $1.8 \times 0.8\,\mathrm{m}$, the flow rotation period is found to be about 2 s (so the rotation rate is $\omega = 3.1$ s^{-1}) at a radius of $r = 2\,\mathrm{cm}$. The angular momentum per unit mass of this flow is therefore $\omega r^2 \simeq 3.1 \times 0.02 \times 0.02 \simeq 1.2 \times 10^{-3}$ $\mathrm{m}^2\,\mathrm{s}^{-1}$. (Since ω is so much grater than the earth's rotation Ω, we can here neglect the contribution from the latter.) The area of the bath is $1.44\ \mathrm{m}^2$, corresponding (if it were circular) to a radius of $R = 0.68\,\mathrm{m}$. Assuming no motion in the initial state (when the plug is removed) other than the earth's rotation, the absolute angular momentum per unit mass at the perimeter would be just $\Omega R^2 = 7.27 \times 10^{-5} \times 0.68^2 = 3.4 \times 10^{-5}\ \ \mathrm{m}^2\,\mathrm{s}^{-1}$.

(c) *Since angular momentum should be conserved, then if there was indeed no motion at the instant you pulled the plug, the maximum possible angular momentum per unit mass in the drain vortex should be the same as that at the perimeter at the initial instant (since that is where the angular momentum was greatest). Compare your answers and comment on the importance of the Earth's rotation for the drain vortex, and hence comment on the validity of the saying.*

If angular momentum were conserved, and if the water in the bath were completely motionless (relative to the earth) then the angular momentum of the flow could not exceed that value associated with the earth's rotation, evaluated at the perimeter (since that is where the maximum angular momentum, with respect to the drain, would be found). But we have seen (at least in the case considered here) that the value at the drain far exceeds the expected value. Since frictional effects will reduce, and not enhance, the angular momentum as the water flows in towards the drain, the only reasonable explanation is that the water was not stationary when the plug was removed. (In fact, a value of angular momentum equal to that observed near the drain would require an initial azimuthal flow velocity v, at the perimeter, such that $(v+\Omega R)\,R = 1.2\times10^{-3}\,\mathrm{m^2\,s^{-1}}$ whence $v \simeq 2$ mm s^{-1}.)

(d) *In view of your answer to (c), what are your thoughts on Perrot's experiment, GFD Lab VI?*

Obviously, the key to executing Perrot's experiment successfully is to take all precautions to eliminate relative motion in the tank, to ensure that the absolute angular momentum of the fluid is dominated by the earth's rotation. This means, *e.g.*, that the net swirl velocity v near the edge much be small compared with ΩR; for $R = 0.68\,\mathrm{m}$, this requires $|v| \ll 5\times10^{-5}\quad \mathrm{m\,s^{-1}}$. Achieving such a state is a very difficult task.

7. *We specialize Eq.(6.44) to two-dimensional, inviscid ($F = 0$) flow of a homogeneous fluid of density ρ_{ref} thus:*

$$\begin{aligned}\frac{Du}{Dt} + \frac{1}{\rho_{ref}}\frac{\partial p}{\partial x} - fv &= 0\\ \frac{Dv}{Dt} + \frac{1}{\rho_{ref}}\frac{\partial p}{\partial y} + fu &= 0\end{aligned}$$

where $\frac{D}{Dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}$ *and the continuity equation is*

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0.$$

(a) *By eliminating the pressure gradient term between the two momentum equations and making use of the continuity equation, show that the quantity* $\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} + f\right)$ *is conserved following the motion: i.e.*

$$\frac{D}{Dt}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} + f\right) = 0.$$

Taking

$$\frac{\partial}{\partial x}\left[\frac{Dv}{Dt} + \frac{1}{\rho_{ref}}\frac{\partial p}{\partial y} + fu\right] - \frac{\partial}{\partial y}\left[\frac{Du}{Dt} + \frac{1}{\rho_{ref}}\frac{\partial p}{\partial x} - fv\right] = 0$$

differentiating products and collecting terms in the total derivative expressions, noting that $f = f(y)$ and that $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$, yields the result that we seek.

(b) *Convince yourself that*

$$\widehat{\mathbf{z}}.\nabla \times \mathbf{u} = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

(see Appendix A.2.2), i.e. that $\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ *is the vertical component of a vector quantity known as the vorticity,* $\nabla \times \mathbf{u}$*, the curl of the velocity field.*

Forming the curl of the velocity field we have:

$$\nabla\times\mathbf{u} = \begin{vmatrix} \widehat{\mathbf{x}} & \widehat{\mathbf{y}} & \widehat{\mathbf{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix} = \left(\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right), \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right), \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\right)$$

and so we see that the $\widehat{\mathbf{z}}$ component is $\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$, known as the relative vorticity.

The quantity $\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} + f$ is known as the 'absolute' vorticity and is made up of 'relative' vorticity (due to motion relative to the rotating planet) and 'planetary' vorticity, f, due to the rotation of the planet itself.

(c) *By computing the 'circulation' — the line integral of* u *about the rectangular element in the* (x, y) *plane shown in Fig.6.22 — show that:*

$$\frac{\text{circulation}}{\text{area enclosed}} = \begin{array}{c}\text{average normal component} \\ \text{of vorticity}\end{array}.$$

The velocity at the center of the element is $\mathbf{u} = \mathbf{u}(x, y)$. Thus evaluating the circulation by summing the contributions along the southern, eastern, northern and western edges we obtain:

$$\begin{aligned} & \underbrace{\delta x\left(u - \frac{\partial u}{\partial y}\frac{\delta y}{2}\right)}_{\text{south}} + \underbrace{+\delta y\left(v + \frac{\partial v}{\partial x}\frac{\delta x}{2}\right)}_{\text{east}} + \\ & \underbrace{-\delta x\left(u + \frac{\partial u}{\partial y}\frac{\delta y}{2}\right)}_{\text{north}} + \underbrace{-\,\delta y\left(v - \frac{\partial v}{\partial x}\frac{\delta x}{2}\right)}_{\text{west}} \\ = & \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\delta x \delta y \end{aligned}$$

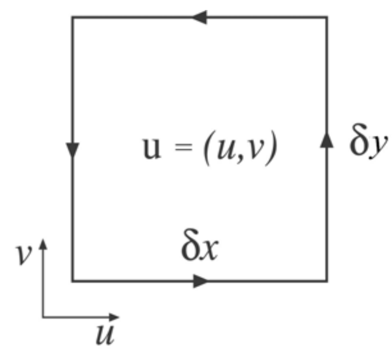


Figure 1: Circulation integral schematic.

Dividing through by the area of the element, $\delta x \delta y$, yields the answer we seek.

Hence deduce that if the fluid element is in solid body rotation then the average vorticity is equal to twice the angular velocity of its rotation.

The above result implies that the vorticity of a fluid in solid body rotation with angular velocity ω is:

$$\omega = \frac{2\pi r V}{\pi r^2} = \frac{2\pi r\,(r\omega)}{\pi r^2} = 2\omega$$

<u>twice</u> the angular velocity of rotation.

(d) *If the tangential velocity in a hurricane varies like* $v = \frac{10^6}{r}\,\mathrm{m\,s^{-1}}$ *where* r *is the radius, calculate the average vorticity between an inner circle of radius* $300\,\mathrm{km}$ *and an outer circle of radius* $500\,\mathrm{km}$*. Express your answer in units of planetary vorticity* f *evaluated at* $20^o N$*. What is the average vorticity within the inner circle?*

The average vorticity between the inner and outer circle is exactly zero because $vr =$ constant is an example of irrotational motion.

The vorticity within the inner circle is: $\frac{2\pi r \times \frac{10^6}{r}\,\mathrm{m\,s^{-1}}}{\pi \times (300\,\mathrm{km})^2} = 2.22 \times 10^{-5}\,\mathrm{s^{-1}}$. The value of f at 20°N is $2 \times 7.27 \times 10^{-5}\,\mathrm{s^{-1}} \sin 20^\circ = 4.9 \times 10^{-5}\,\mathrm{s^{-1}}$. Thus the hurricane has an average vorticity of 0.45 times the local Coriolis parameter.