Atmosphere, Ocean and Climate Dynamics
Answers to Chapter 5

1. *Fig.5.5 shows the net incoming solar and outgoing longwave irradiance at the top of the atmosphere. Note that there is a net gain of radiation in low latitudes and a net loss in high latitudes. By inspection of the figure, estimate the magnitude of the poleward energy flux that must be carried by the atmosphere-ocean system across the* $30°$ *latitude circle, to achieve a steady state.*

   The energy absorbed between the equator and $30°$ latitude is, roughly, $\frac{1}{2} \times 50\mathrm{W\,m^{-2}} \times \frac{1}{4} \times 5 \times 10^{14}\,\mathrm{m}^2 = 3.125 \times 10^{15}\mathrm{W}$, roughly comparable to Fig.5.6. [Note $30°$ latitude divides the hemisphere in to equal areas.]

2. *Suppose that the Earth's rotation axis were normal to the Earth-Sun line. The solar flux, measured per unit area in a plane normal to the Earth-Sun line, is* S_0. *By considering the solar flux incident on a latitude belt bounded by latitudes* $(\varphi, \varphi + d\varphi)$, *show that* F, *the 24hr-averaged solar flux per unit area of the Earth's surface, varies with latitude as*

   $$F = \frac{S_0}{\pi} \cos\varphi \, .$$

   The area of parallel solar beam intercepted by latitudes φ and $\varphi + d\varphi$ is $2a\cos\varphi \times dy$ where $dy = ad\varphi \cos\varphi$. The surface area of the planet between latitudes φ and $\varphi + d\varphi$ is $2\pi a \cos\varphi \times ad\varphi$. Thus:

   $$F = \frac{2a\cos\varphi \times dy \times S_0}{2\pi a \cos\varphi \times ad\varphi} = \frac{2a\cos\varphi \times ad\varphi \cos\varphi \times S_0}{2\pi a^2 \cos\varphi \times d\varphi} = \frac{S_0}{\pi}\cos\varphi.$$

   (a) *Using this result, suppose that the atmosphere is completely transparent to solar radiation, but opaque to infrared such that, separately at each latitude, the radiation budget can be represented as for a "single slab" atmosphere — the "single slab" model is discussed in Section 2.3.1. Determine how surface temperature varies with latitude.*

   Balance at top of the atmosphere in each latitude belt:

   $$(1 - \alpha_p)\,\frac{S_0}{\pi}\cos\varphi = A \uparrow = \sigma T_a^4$$

Balance at ground in each latitude belt:

$$(1-\alpha_p)\frac{S_0}{\pi}\cos\varphi + A\downarrow = \sigma T_s^4$$

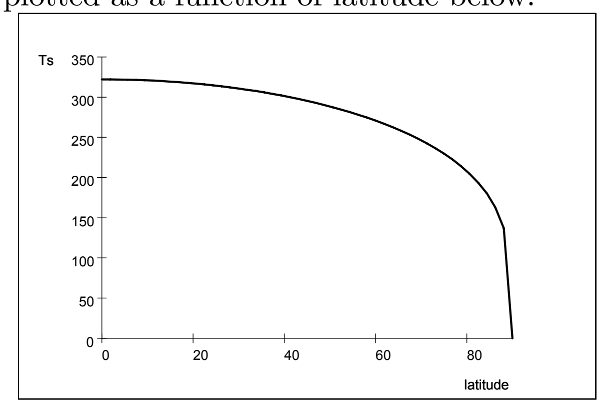
But $A\uparrow = A\downarrow = \sigma T_a^4$ hence:

$$2\,(1-\alpha_p)\frac{S_0}{\pi}\cos\varphi = \sigma T_s^4$$

and so

$$T_s = \left[\frac{8\cos\varphi}{\pi}\right]^{\frac{1}{4}}\left[\frac{(1-\alpha_p)\,S_0}{4\sigma}\right]^{\frac{1}{4}} = \left[\frac{8\cos\varphi}{\pi}\right]^{\frac{1}{4}} T_e$$

where T_e is given by Eq.(2.4).
The solution is plotted as a function of latitude below.



(b) *Calculate the surface temperature at the equator, 30°, and 60° latitude, using data for Earth albedo and S_0.*

$T_s(0°) = \left[\frac{8\cos 0°}{\pi}\right]^{\frac{1}{4}} 255 = 322.13$
$T_s(30°) = \left[\frac{8\cos 30°}{\pi}\right]^{\frac{1}{4}} 255 = 310.75$
$T_s(60°) = \left[\frac{8\cos 60°}{\pi}\right]^{\frac{1}{4}} 255 = 270.87$

3. *Use the hydrostatic relation and the equation of state of an ideal gas to show that the* $1000 - 500\,\mathrm{mbar}$ *"thickness",* $\Delta z = z(500\,\mathrm{mbar}) -$

$z(1000\,\mathrm{mbar})$ *is related to the mean temperature* $\langle T\rangle$ *of the* $1000-500\,\mathrm{mbar}$ *layer by*

$$\Delta z = \frac{R\,\langle T\rangle}{g}\ln 2\ ,$$

where

$$\langle T\rangle = \frac{\int T\ d\ln p}{\int d\ln p}\ ,$$

where the integrals are from $500\,\mathrm{mbar}$ *to* $1000\,\mathrm{mbar}$*. (Note that* $1000\,\mathrm{mbar} \equiv 1000h\,\mathrm{Pa} \equiv 10^5\,\mathrm{Pa}$*).*

Combining hydrostatic balance and the ideal gas law, we can write:

$$\frac{\partial p}{\partial z} = -\frac{gp}{RT}$$

whence

$$\frac{\partial z}{\partial p} = -\frac{RT}{gp}\ .$$

The height of a given pressure surface is dependent on the surface pressure p_s and the average temperature below that pressure surface:

$$z(p) = \frac{R}{g}\int_p^{p_s} T\frac{dp}{p} \ = \frac{R}{g}\int_p^{p_s} T d\ln p = \frac{R}{g}\,\langle T\rangle \int_p^{p_s} d\ln p,$$

using the definition of $\langle T\rangle$ given in the question.

Thus $\Delta z = z(500\mathrm{h\,Pa}) - z(1000\mathrm{h\,Pa})$ is given by:

$$\Delta z = \frac{R\,\langle T\rangle}{g}\ln 2.$$

a. *Compute the thickness of the surface to* $500\,\mathrm{mbar}$ *layer at* 30° *and* 60° *latitude assuming that the surface temperatures computed in Q2b. above extend uniformly up to* $500\,\mathrm{mbar}$*.*

   $\Delta z = \frac{287\times 310.75}{9.81}\ln 2 = 6301.6$ at 30°

   $\Delta z = \frac{270.87\times 310.75}{9.81}\ln 2 = 5947.4$ at 60°

b. *Figs.7.4 and 7.25 (of Chapter 7) show* $500\,\mathrm{mbar}$ *and surface pressure analyses for 12GMT on June 21, 2003. Calculate* $\langle T\rangle$ *for the* $1000\,\mathrm{mbar}$ *to* $500\,\mathrm{mbar}$ *layer at the center of the* $500\,\mathrm{mbar}$ *trough at*

$50°$N, $120°$W and at the center of the ridge at $40°$N, $90°$W. [N.B. You will need to convert from surface pressure, p_s, to height of the 1000h Pa *surface, z_{1000}; to do so use the (approximate) formula*

$$z_{1000} \cong 10 \ (p_s - 1000) \ ,$$

*where z_{1000} is in meters and p_s is in $h\,$*Pa.*] Is $\langle T \rangle$ greater in the ridge or the trough? Comment on and physically interpret your result.*

We know that:

$$\langle T \rangle = \frac{g\Delta z}{R\ln 2}$$

In the trough: $p_s = 1008\text{h Pa}$ and so $z_{1000} \cong 10 \ (1008 - 1000) = 80\,\text{m}$; $z_{500} = 5520\,\text{m}$.

Thus $\Delta z = 5520 - 80 = 5440\,\text{m}$ and so $\langle T \rangle = \frac{9.81 \times 5440}{287 \times \ln 2} K = 268\,\text{K}$.

In the ridge: $p_s = 1016\text{h Pa}$ and so $z_{1000} \cong 10 \ (1016 - 1000) = 160\,\text{m}$; $z_{500} = 5820\,\text{m}$

Thus $\Delta z = 5820 - 160 = 5660\,\text{m}$ and so $\langle T \rangle = \frac{9.81 \times 5660}{287 \times \ln 2} K = 279\,\text{K}$.

The ridge, then, is some 11 K warmer than the trough. This is reasonable since in the ridge (trough) air has pushed from the south (north) and is thus warm (cold).

4. *Use the expression for saturated specific humidity, Eq.(4.24) and the empirical relation for saturated vapor pressure $e_s(T)$, Eq.(1.4) (where $A = 6.11\,\text{mbar}$ and $\beta = 0.067\ ^\circ\text{C}^{-1}$ and T is in $^\circ$C) to compute from the graph of $T(p)$ in the tropical belt shown in Fig.4.9, vertical profiles of saturated specific humidity, $q^*(p)$. You will need to look up values of R and R_v from Chapter 1.*

   Compare your q^* profiles with observed profiles of q in the tropics shown in Fig.5.15. Comment?

   We evaluate:

   $q_* = \left(\frac{R}{R_v}\right) \frac{e_s(T)}{p}$ where $R_v = 461.39$ J kg^{-1} K^{-1}; $R = 287.05$ J kg^{-1} K^{-1} with $e_s = A \exp(\beta T)$ where $A = 6.11\,\text{mbar}$ and $\beta = 0.067\ ^\circ\text{C}^{-1}$.

   Thus, evaluating the various numerical factors, we have:

   $q_* = 0.622\,14 \times \frac{6.11 \exp(0.067 \times T\,^\circ\text{C})}{p(\,\text{mbar})}$

From observations of $T(p)$ at the equator, we evaluate q_* from the above equation and compare to observations of q in the following table.

p	T(equator) in °C	q_* (g/kg)	q (g/kg)
1000	25.6508	21.2	16.49
950	22.6972	18.3	14.56
900	19.9835	16.2	12.52
850	17.6248	14.6	10.67
700	9.33311	10.1	6.04
500	-5.59631	5.2	2.26
400	-15.9016	3.3	1.00
300	-31.2027	1.56	0.31
200			
100			

Note that at each level in the atmosphere, $q \lesssim q_*$: the air is less than, but reasonably close to saturation at each level.