Atmosphere, Ocean and Climate Dynamics
Answers to Chapter 1

1. *Given that the acceleration due to gravity decays with height from the centre of the earth following an inverse square law, what is the percentage change in g from the earth's surface to an altitude of $100\,\mathrm{km}$? (See also Q.6 of Chapter 3.*

   Writing $g = \frac{\mathrm{const}}{r^2}$, $\delta g = -2\frac{g\delta r}{r}$ and so:

$$\frac{\delta g}{g} = -2\frac{\delta r}{r}$$

   Percentage change in g from the earth's surface to an altitude of $100\,\mathrm{km}$ is then $\sim 2 \times \frac{10^5}{6\times10^6} \sim 3\%$.

2. *Compute the mean pressure at the earth's surface given the total mass of the atmosphere, M_a (Table 1.3), the acceleration due to gravity, g, and the radius of the earth, a (Table 1.1).*

   The pressure at the earth's surface is the mass of the atmosphere divided by the surface area of the earth: $p_s = \frac{gM}{4\pi a^2}$. Inserting numbers from the Table we find that $p_s = \frac{9.81\times5.26\times10^{18}}{5.09\times10^{14}} = 1.0138 \times 10^5\,\mathrm{Pa}$.

3. *Express your answer to Q.2 in terms of the number of apples per square meter required to exert the same pressure. You may assume that a typical apple weighs $0.2\,\mathrm{kg}$. If the average density of air is 5 apples per m^3 (in apple units) calculate how high the apples would have to be stacked at this density to exert a surface pressure equal to $1000h\,\mathrm{Pa}$. Compare your estimate to the scale height, H, given by Eq.(3.6) in Section 3.3.*

   Let the number of apples per square meter be n, each of mass m, then: $n \times m \times g = p_s$ or, rearranging:

$$n = \frac{p_s}{m \times g}.$$

   Inserting numbers we find that $n = 50,000$ apples. If the average density of air is 5 apples per m^3 then the column must be $10\,\mathrm{km}$ in height. This is roughly equal to the scale height of the atmosphere of $7\,\mathrm{km}$ or so.

4. *Using (i) Eq.(1.4), which relates the saturation vapor pressure of* H_2O *to temperature* T*, and (ii) the equation of state of water vapor* $e = \rho_v R_v T$ *— see discussion in Section 1.3.2 — compute the maximum amount of water vapor per unit volume that air can hold at the surface, where* $T_s = 288\,\mathrm{K}$*, and at a height of* 10 km *where (from Fig.3.1)* $T_{10\ \mathrm{km}} = 220\,\mathrm{K}$*. Express your answer in* $\mathrm{kg\,m^{-3}}$*. What are the implications of your results for the distribution of water vapor in the atmosphere?*

   Form Eq.(1.4) we deduce that at the surface
   $e_s = 6.11 \times 10^2 \times \exp\left(0.067 \times (288 - 273)\right) = 1669.2 = 16.7\mathrm{h\,Pa}$.

   This implies a density at saturation of $\rho_v = \frac{e_s}{R_v T} = \frac{1670}{461 \times 288} = 1.26 \times 10^{-2}\,\mathrm{kg\,m^{-3}}$.

   At a height of 10 km, $e_s = 6.11 \times 10^2 \times \exp\left(0.067 \times (220 - 273)\right) = 0.18\mathrm{h\,Pa}$ and $\rho_v = \frac{18}{461 \times 220}\,\mathrm{kg\,m^{-3}} = 0.02 \times 10^{-2}\,\mathrm{kg\,m^{-3}}$, a very small amount of water vapor. We conclude that upper levels of the atmosphere are so cold that they hold very little water vapor.