Chapter 3

Elementary Applications of the Basic Equations

3.1. An aircraft flying a heading of 60° (i.e., 60° to the east of north) at air speed 200 m s^{-1} moves relative to the ground due east (90°) at 225 m s^{-1}. If the plane is flying at constant pressure, what is its rate of change in altitude (in meters per kilometer horizontal distance) assuming a steady pressure field, geostrophic winds, and $f = 10^{-4}$ s^{-1}?

Solution: For geostrophic motion, $(\partial Z/\partial x)_p = f v_g/g$. Thus, assuming that the wind is geostrophic, we need the y component to compute the change in height for isobaric flight in the x direction. But from Figure 3.1, $\mathbf{V}_{\text{wind}} = \mathbf{V}_{\text{ground}} - \mathbf{V}_{\text{air}}$, so that $v_g = \mathbf{j} \cdot \mathbf{V}_{\text{wind}} = \mathbf{j} \cdot \mathbf{V}_{\text{air}} = -|\mathbf{V}_{\mathbf{air}}| \cos(\pi/3)$, Now $|\mathbf{V}_{\text{air}}| = 200$ m s^{-1}; thus, $v_g = -100$ m s^{-1}, and $\left(\frac{\partial Z}{\partial x}\right)_p = \frac{f v_g}{g} = \frac{(10^{-4})(-100)}{9.8} \approx -1$ m/km

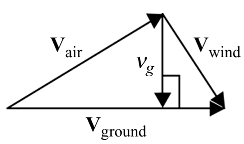

FIGURE 3.1

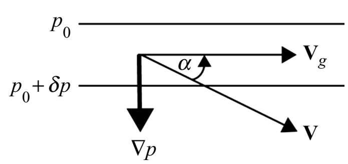

FIGURE 3.2

3.2. The actual wind is directed 30° to the right of the geostrophic wind. If the geostrophic wind is 20 m s^{-1}, what is the rate of change of wind speed? Let $f = 10^{-4}$ s^{-1}.

Solution: From (3.9) $DV/Dt = -\partial\Phi/\partial s$, where $\partial\Phi/\partial s$ is the component of $\nabla\Phi$ that is parallel to the velocity $\mathbf{V}$. Thus, from Figure 3.2, $\partial\Phi/\partial s = |\nabla\Phi| \sin(\pi/6)$. But $|\nabla\Phi| = f\left|\mathbf{V}_g\right|$. Thus, $DV/Dt = -\left(10^{-4}\right)(20)(0.5) = -10^{-3}$ m s^{-2}.

3.3. A tornado rotates with constant angular velocity ω. Show that the surface pressure at the center of the tornado is given by $p = p_0 \exp\left(\frac{-\omega^2 r_0^2}{2RT}\right)$, where p_0 is the surface pressure at a distance r_0 from the center and T is the temperature (assumed constant). If the temperature is 288 K, and pressure and wind speed at 100 m from the center are 1000 hPa and 100 m s^{-1}, respectively, what is the central pressure?

Solution: Letting r be the distance from the axis of rotation of the tornado, and shifting to height coordinates, the cyclostrophic balance (3.14) can be expressed as $\frac{v^2}{r} = \frac{1}{\rho}\frac{\partial p}{\partial r}$. But $v = \omega r$ and $\rho^{-1} = RT/p$, so that $\frac{\partial \ln p}{\partial r} = \frac{\omega^2 r}{RT}$. Integrating with respect to r: $\int_{p_0}^{p} d\ln p = \left(\frac{\omega^2}{RT}\right)\int_{r_0}^{0} r dr$, or $p = p_0 \exp\left[-\frac{\omega^2 r_0^2}{2RT}\right]$. Since $\frac{\omega^2 r_0^2}{2RT} \ll 1$, the central pressure is given by $p \approx p_0\left(1 - \frac{\omega^2 r_0^2}{2RT}\right) = 940$ hPa.

3.4. Calculate the geostrophic wind speed ($\mathrm{m\,s^{-1}}$) on an isobaric surface for a geopotential height gradient of 100 m per 1000 km and compare with all possible gradient wind speeds for the same geopotential height gradient and a radius of curvature of ± 500 km. Let $f = 10^{-4}\,\mathrm{s^{-1}}$.

Solution: From (3.11) $V_g = -f^{-1}(\partial\Phi/\partial n) = -(g/f)(\partial Z/\partial n)$. Thus, $V_g = \pm\left(9.8/10^{-4}\right)\left(100/10^{6}\right) = \pm 9.8\,\mathrm{m\,s^{-1}}$. (Note that the geostrophic speed can be positive or negative, while the gradient wind speed must always be positive.) From (3.15) $V_{grad} = -\left(\frac{fR}{2}\right) \pm \left(\frac{f^2R^2}{4} + fRV_g\right)^{1/2} = \pm 25 \pm (625 \pm 490)^{1/2}$. Noting that the gradient wind must be positive for all permitted solutions: $V_{grad} = 8.4$ m/s regular low; 13.4 m/s regular high; 58.4 m/s anomalous low; and 36.6 m/s anomalous high.

3.5. Determine the maximum possible ratio of the normal anticyclonic gradient wind speed to the geostrophic wind speed for the same pressure gradient.

Solution: From Table 3.1, a regulary high takes the negative root in (3.15) and has $R < 0$, and $V_{grad} = -\left(\frac{fR}{2}\right) - \left(\frac{f^2R^2}{4} + fRV_g\right)^{1/2}$. Thus, $\left(V_{grad}\right)_{\max} = -\left(\frac{fR}{2}\right)$; that is, $\frac{f^2R^2}{4} + fRV_g = 0$ for the maximum gradient wind speed. Thus, for the maximum gradient wind, $V_g = -fR/4 = (1/2)\left(V_{grad}\right)_{\max}$ so that maximum gradient wind is twice the geostrophic wind speed in a normal anticyclone.

3.6. Show that the geostrophic balance in isothermal coordinates may be written

$$f\mathbf{V}_g = \mathbf{k} \times \nabla_T \left(RT \ln p + \Phi\right).$$

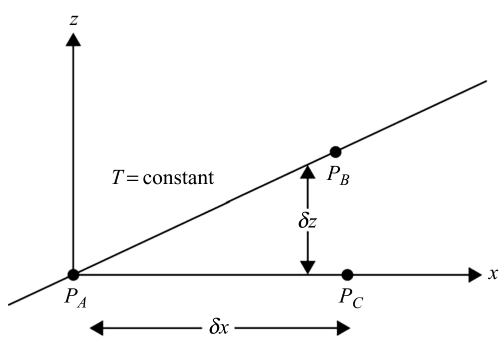

FIGURE 3.6

Solution: Now from the figure: $\frac{(p_B - p_A)}{\delta x} = \frac{(p_C - p_A)}{\delta x} + \frac{(p_B - p_C)}{\delta Z}\left(\frac{\delta Z}{\delta x}\right)_T$. Thus: $\left(\frac{\partial p}{\partial x}\right)_T = \left(\frac{\partial p}{\partial x}\right)_Z + \left(\frac{\partial p}{\partial Z}\right)\left(\frac{\partial Z}{\partial x}\right)_T$ But: $fv_g = \frac{1}{\rho}\left(\frac{\partial p}{\partial x}\right)_Z = \frac{1}{\rho}\left(\frac{\partial p}{\partial x}\right)_T + g_0\left(\frac{\partial Z}{\partial x}\right)_T$, where we have used $\frac{\partial p}{\partial Z} = -\rho g_0$. If we eliminate ρ using $p = \rho RT$, and noting that $\Phi = g_0 Z$, we obtain $fv_g = \frac{RT}{p}\left(\frac{\partial p}{\partial x}\right)_T + \left(\frac{\partial \Phi}{\partial x}\right)_T = \frac{\partial}{\partial x}\left(RT\ln p + \Phi\right)_T$ with analogous expression for fu_g.

3.7. Determine the radii of curvature for the trajectories of air parcels located 500 km to the east, north, south, and west of the center of a circular low-pressure system, respectively. The system is moving eastward at $15\,\mathrm{m\,s^{-1}}$. Assume geostrophic flow with a uniform tangential wind speed of $15\,\mathrm{m\,s^{-1}}$.

Solution: From (3.24) $R_s = R_t\left(1 - c\cos\gamma/V\right)$, where $c = V = 15$ m/s, and $R_s = 500$ km. Thus,

North of center	$R_t = R_s/2 = 250\,\mathrm{km}\ (\gamma = \pi)$
West of center	$R_t = R_s = 500\,\mathrm{km}\ (\gamma = 3\pi/2)$
South of center	$R_t \to \infty\ (\gamma = 0)$
East of center	$R_t = R_s = 500\,\mathrm{km}\ (\gamma = \pi/2)$

3.8. Determine the normal gradient wind speeds for the four air parcels of Problem 3.7 using the radii of curvature computed in Problem 3.7. Compare these speeds with the geostrophic speed. (Let $f = 10^{-4}\,\mathrm{s}^{-1}$.) Use the gradient wind speeds calculated here to recompute the radii of curvature for the four air parcels referred to in Problem 3.7. Use these new estimates of the radii of curvature to recompute the gradient wind speeds for the four air parcels. What fractional error is made in the radii of curvature by using the geostrophic wind approximation in this case? (Note that further iterations could be carried out, but would rapidly converge.)

Solution: Gradient wind speed for normal low is given by

$$V_{grad} = -\left(\frac{fR_t}{2}\right) + \left[\left(\frac{fR_t}{2}\right)^2 + fR_t V_g\right]^{1/2}.$$

Using the radii of curvature from Problem 3.7, we obtain

North of center $V_{grad} = 10.5$ m/s
West of center $V_{grad} = 12.1$ m/s
South of center $V_{grad} = 15$ m/s
East of center $V_{grad} = 12.1$ m/s

Substituting these values of gradient wind into the formula of Problem 3.7, we find that only the trajectory curvature north of the center changes: North of center $R_t = R_s/(1 + 15/10.5) = 206$ km. Plugging this value into the gradient wind formula, we find North of center $V_{grad} = 10.07$ m/s, which is about a 4% decrease.

3.9. Show that as the pressure gradient approaches zero, the gradient wind reduces to the geostrophic wind for a normal anticyclone and to inertial flow (Section 3.2.3) for an anomalous anticyclone.

Solution: For $\nabla p \to 0$, $V_g \to 0$. But $V_{grad} = -\left(\frac{fR}{2}\right) \pm \left(\frac{fR}{2}\right)\left[1 + \frac{4V_g}{fR}\right]^{1/2}$. For $4V_g \ll fR$ the above simplifies to $V_{grad} \approx +\left(\frac{fR}{2}\right)\left[-1 \pm \left(1 + \frac{2V_g}{fR}\right)\right]$, where we have used the expansion $(1+x)^{1/2} \approx 1 + x/2$, valid for small x. The positive root gives $V_{grad} = V_g$; the negative root gives $V_{grad} \approx -fR$ for $V_g \to 0$.

3.10. The mean temperature in the layer between 750 and 500 hPa decreases eastward by 3°C per 100 km. If the 750 hPa geostrophic wind is from the southeast at $20\,\mathrm{m\,s^{-1}}$, what is the geostrophic wind speed and direction at 500 hPa? Let $f = 10^{-4}\,\mathrm{s}^{-1}$.

Solution: From (3.33)
$$u_{500} = u_{750} + u_T = -14.1 + u_T \ \text{m/s}$$
$$v_{500} = v_{750} + v_T = +14.1 + v_T \ \text{m/s}$$

But, $u_T = 0$; and $v_T = \frac{R}{f}\left(\frac{\partial T}{\partial x}\right)_p \ln\frac{750}{500} = -34.5$ m/s.

Thus, $\mathbf{V_g}\,(500) = (-14.1, -20.4)$ m/s; or 25 m/s speed from 34° east of north.

3.11. What is the mean temperature advection in the 750-500 hPa layer in Problem 3.10?

Solution: Mean advection $= -\mathbf{V}\cdot\nabla\mathbf{T} = -\overline{u}\partial\overline{T}/\partial x$, where $\overline{u} = (u_{500} + u_{750})/2$ and $\partial\overline{T}/\partial x = -3\times10^{-5}$°C/m. Thus, $-\overline{u}\partial\overline{T}/\partial x = -4.23\times10^{-4}$°C/s, (−1.5°C/hr).

3.12. Suppose that a vertical column of the atmosphere at 43°N is initially isothermal from 900 to 500 hPa. The geostrophic wind is $10\,\mathrm{m\,s^{-1}}$ from the south at 900 hPa, $10\,\mathrm{m\,s^{-1}}$ from the west at 700 hPa, and $20\,\mathrm{m\,s^{-1}}$ from the west at 500 hPa. Calculate the mean horizontal temperature gradients in the two layers 900-700 hPa and 700-500 hPa. Compute the rate of advective temperature change in each layer. How long would this advection pattern have to persist in order to establish a dry adiabatic lapse rate between 600 and 800 hPa? (Assume that the lapse rate is constant between 900 and 500 hPa, and that the 800-600 hPa layer thickness is 2.25 km.)

Solution: The wind components in tabular form are:

p (hPa)	u_g m/s	v_g m/s	u_T m/s	v_T m/s
900	0	10		
			10	−10
700	10	0		
			10	0
500	20	0		

From (3.32) $\frac{\partial \overline{T}}{\partial y} = -\left(\frac{fu_T}{R\ln(p_0/p_1)}\right)$; $\frac{\partial \overline{T}}{\partial x} = +\left(\frac{fv_T}{R\ln(p_0/p_1)}\right)$. For layer 900-700 hPa: $\partial T/\partial x = \partial T/\partial y = -(1.39°\text{C})/(100\,\text{km})$. For layer 700-500 hPa: $\partial T/\partial x = 0$; $\partial T/\partial y = -(1.03°\text{C})/(100\,\text{km})$. The advective rate of temperature change is $\frac{\partial \overline{T}}{\partial t} = -\overline{u}\frac{\partial \overline{T}}{\partial x} - \overline{v}\frac{\partial \overline{T}}{\partial y}$. The average wind in the 900-700 hPa layer is $\overline{u} = \overline{v} = 5$ m/s, and in the 700-500 hPa layer it is $\overline{u} = 15$ m/s, $\overline{v} = 0$. Thus, for the 900-700 hPa layer $\frac{\partial \overline{T}}{\partial t} = 1.39 \times 10^{-4}$°C/s, or about 0.5°C/hr; for the 700-500 hPa layer, $\partial \overline{T}/\partial t = 0$. Thus, the temperature difference between 800 hPa and 600 hPa increases by 0.5°C/hr. Assuming a 2.25 km thickness, we find that an adiabatic lapse rate (9.8°C/km) would be established in about 44 hours.

3.13. An airplane pilot crossing the ocean at 45°N latitude has both a pressure altimeter and a radar altimeter, the latter measuring his absolute height above the sea. Flying at an air speed of $100\,\text{m}\,\text{s}^{-1}$, he maintains altitude by referring to his pressure altimeter set for a sea-level pressure of 1013 hPa. He holds an indicated 6000 m altitude. At the beginning of a one-hour period, he notes that his radar altimeter reads 5700 m, and at the end of the hour he notes that it reads 5950 m. In what direction and approximately how far has he drifted from his heading?

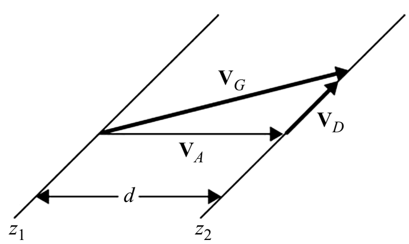

FIGURE 3.13

Solution: Let $\mathbf{V}_A$ be the velocity of the plane with respect to the air, $\mathbf{V}_G$ be the velocity of the plane relative to the ground, and $\mathbf{V}_D$ be the wind velocity. Assume that the wind is geostrophic so is parallel to lines of constant height on an isobaric surface. The height change of the plane in distance $d = |\mathbf{V}_A|\,t$ (where t is the elapsed time) is proportional to the component of the wind $\perp$ to $\mathbf{V}_A$. For convenience assume that the heading (hence $\mathbf{V}_A$) is eastward. Then the northward component of the geostrophic wind is $v_g = (g/f)\,\partial Z/\partial x \approx (g/f)\,(Z_2 - Z_1)/d$. The drift of the plane from its heading is $D = v_g t$. Thus,
$D = \left(\frac{g}{f}\right)\frac{(Z_2-Z_1)}{|\mathbf{V}_A|} = \left(\frac{9.8}{1.03\times10^{-4}}\right)\left(\frac{250}{100}\right) = 238$ km
The drift is to the left of the heading in the Northern Hemisphere.

3.14. Work out a gradient wind classification scheme equivalent to Table 3.1 for the Southern Hemisphere ($f < 0$) case.

	$R > 0$	$R < 0$
$\frac{\partial\Phi}{\partial n} > 0$	+ root: ($V > -fR/2$) anomalous high − root: ($V < -fR/2$) regular high	+ root: regular low − root: unphysical
$\frac{\partial\Phi}{\partial n} < 0$	+ root: antibaric flow (anomalous low) − root: unphysical	+ root: unphysical − root: unphysical

3.15. In the geostrophic momentum approximation (Hoskins, 1975), the gradient wind formula for steady circular flow (3.17) is replaced by the approximation $VV_gR^{-1} + fV = fV_g$. Compare the speeds V computed using this approximation with those obtained in Problem 3.8 using the gradient wind formula.

Solution: From the above formula, $V = V_g\left[1 + V_g/(fR)\right]^{-1}$. North of low center $R = 250\,\text{km}$, so $V = 9.38\,\text{m/s}$. East and west of low $R = 500\,\text{km}$, so $V = 11.54\,\text{m/s}$. South of low $R \to \infty$, so $V = 15\,\text{m/s}$. For finite positive R the gradient wind is underestimated in this approximation.

3.16. How large can the ratio $V_g/(fR)$ be before the geostrophic momentum approximation differs from the gradient wind approximation by 10% for cyclonic flow?

Solution: The percentage error can be computed by noting that for a regular low $V_{grad} = (fR/2)\left[-1 + \left(1 + 4V_g/fR\right)^{1/2}\right]$, while the approximation of Problem 3.15 can be written as $V = (fR/2)\left(2V_g/fR\right)\left[1 + V_g/(fR)\right]^{-1}$. By trial and error it can be shown that for $V_g/fR = 1/2$, $V = 0.667\,(fR/2)$, and $V_{grad} = 0.732\,(fR/2)$, so that V is about a 10% underestimate.

3.17. The planet Venus rotates about its axis so slowly that to a reasonable approximation the Coriolis parameter may be set equal to zero. For steady, frictionless motion parallel to latitude circles, the momentum equation (2.20) then reduces to a type of cyclostrophic balance: $\frac{u^2\tan\phi}{a} = -\frac{1}{\rho}\frac{\partial p}{\partial y}$. By transforming this expression to isobaric coordinates show that the thermal wind equation in this case can be expressed in the form $\omega_r^2\,(p_1) - \omega_r^2\,(p_0) = \frac{-R\,\ln\,(p_0/p_1)}{(a\sin\phi\cos\phi)}\frac{\partial\langle T\rangle}{\partial y}$, where R is the gas constant, a is the radius of the planet, and $\omega_r \equiv u/(a\cos\phi)$ is the relative angular velocity. How must $\langle T\rangle$ (the vertically averaged temperature) vary with respect to latitude in order for ω_r to be a function only of pressure? If the zonal velocity at about 60 km height above the equator ($p_1 = 2.9 \times 10^5$ Pa) is $100\,\text{m s}^{-1}$ and the zonal velocity vanishes at the surface of the planet ($p_0 = 9.5 \times 10^6$ Pa), what is the vertically averaged temperature difference between the equator and pole, assuming that ω_r depends only on pressure? The planetary radius is $a = 6100\,\text{km}$, and the gas constant is $R = 187\,\text{J kg}^{-1}\,\text{K}^{-1}$.

Solution: Now the cyclostrophic balance can be rewritten as $\frac{u^2\tan\phi}{a} = \omega_r^2 a\cos\phi\sin\phi = -\left(\frac{\partial\Phi}{\partial y}\right)_p$. But since $\frac{\partial\Phi}{\partial p} = -\frac{RT}{p}$, differentiating in p gives $\frac{\partial\omega_r^2}{\partial p} = \frac{1}{a\cos\phi\sin\phi}\frac{\partial}{\partial y}\left(\frac{RT}{p}\right)$. Integrating in p and defining a layer average temperature $\langle T\rangle$ gives $\omega_r^2\,(p_1) - \omega_r^2\,(p_0) = -\frac{R\ln(p_0/p_1)}{a\cos\phi\sin\phi}\frac{\partial\langle T\rangle}{\partial y}$. For angular velocity to depend only on pressure requires $\frac{\partial\langle T\rangle}{\partial y} \propto \sin\phi\cos\phi$, or since $dy = ad\phi$ $\langle T\rangle = T_e - T'\sin^2\phi$, where T_e is a constant, and $T' = \frac{\left[\omega_r^2(p_1) - \omega_r^2(p_0)\right]a^2}{2R\ln(p_0/p_1)} = \frac{100^2}{(2)(187)(\ln(9.5/0.29))} = 7.66\,\text{K}$.

3.18. Suppose that during the passage of a cyclonic storm the radius of curvature of the isobars is observed to be +800 km at a station where the wind is veering (turning clockwise) at a rate of 10° per hour. What is the radius of curvature of the trajectory for an air parcel that is passing over the station? (The wind speed is $20\,\text{m s}^{-1}$.)

Solution: From equation (3.23) $R_t = V\left(\partial\beta/\partial t + V/R_s\right)^{-1}$. But $V = 20\,\text{m/s}$, $R_s = 8 \times 10^5\,\text{m}$, and $\partial\beta/\partial t = -(\pi/18)\,(1/3600)\ \text{rad s}^{-1}$. Thus, $R_t = -852\,\text{km}$.

3.19. Show that the divergence of the geostrophic wind in isobaric coordinates on the spherical earth is given by

$$\nabla\cdot\mathbf{V}_g = -\frac{1}{fa}\frac{\partial\Phi}{\partial x}\left(\frac{\cos\phi}{\sin\phi}\right) = -v_g\left(\frac{\cot\phi}{a}\right)$$

(Use the spherical coordinate expression for the divergence operator given in Appendix C.)

Solution: Now in spherical coordinates; $\nabla\cdot\mathbf{V}_g = \frac{1}{a\cos\phi}\left[\frac{\partial u_g}{\partial\lambda} + \frac{\partial(v_g\cos\phi)}{\partial\phi}\right]$, but $fv_g = \left(\frac{1}{a}\right)\left(\frac{\partial\Phi}{\cos\phi\partial\lambda}\right)$; $fu_g = -\left(\frac{1}{a}\right)\left(\frac{\partial\Phi}{\partial\phi}\right)$ Thus,

$$\nabla\cdot\mathbf{V}_g = \frac{1}{a\cos\phi}\frac{\partial(1/f)}{\partial\phi}\frac{\partial\Phi}{a\partial\lambda} = \frac{f}{a}\frac{\partial\,(1/f)}{\partial\phi}\left(v_g\right) = -\frac{1}{af}\frac{\partial(f)}{\partial\phi}\left(v_g\right) = -v_g\left(\frac{\cot\phi}{a}\right).$$

3.20. The following wind data were received from 50 km to the east, north, west, and south of a station, respectively: 90°, $10\,\mathrm{m\,s^{-1}}$; 120°, $4\,\mathrm{m\,s^{-1}}$; 90°, $8\,\mathrm{m\,s^{-1}}$; 60°, $4\,\mathrm{m\,s^{-1}}$. Calculate the approximate horizontal divergence at the station.

Solution: Let $\delta x = \delta y = 50$ km. Then, letting A, B, C, D designate the points east, north, west, and south of the station, respectively, we obtain the finite difference formula:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \approx \frac{u_A - u_C}{2\delta x} + \frac{v_B - v_D}{2\delta y} = \frac{(-10+8)}{10^5} + \frac{(2+2)}{10^5} = 2 \times 10^{-5}\ \mathrm{s^{-1}}.$$

3.21. Suppose that the wind speeds given in Problem 3.20 are each in error by ±10%. What would be the percent error in the calculated horizontal divergence in the worst case?

Solution: For the worst case, let $u_A = -9\,\mathrm{m\,s^{-1}}$, $u_C = -8.8\,\mathrm{m\,s^{-1}}$, $v_B = 2.2\,\mathrm{m\,s^{-1}}$, $v_D = -2.2\,\mathrm{m\,s^{-1}}$. Then $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \frac{(-9+8.8)}{10^5} + \frac{(2.2+2.2)}{10^5} = 4.2 \times 10^{-5}\ \mathrm{s^{-1}}$, which is a 110% error.

3.22. The divergence of the horizontal wind at various pressure levels above a given station is shown in the following table.

Pressure (hPa)	$\nabla \cdot \mathbf{V}$ $(\times 10^{-5}\,\mathrm{s^{-1}})$
1000	+0.9
850	+0.6
700	+0.3
500	0.0
300	−0.6
100	−1.0

Compute the vertical velocity at each level assuming an isothermal atmosphere with temperature 260 K and letting $w = 0$ at 1000 hPa.

Solution: From eq. (3.38): $\omega(p_1) = \omega(p_0) + (p_0 - p_1)\langle \nabla \cdot \mathbf{V}\rangle$, where in this case the vertical average of the divergence in each layer must be estimated from averaging the top and bottom values: $\langle \nabla \cdot \mathbf{V}\rangle = \frac{1}{2}\left[(\nabla \cdot \mathbf{V})_{p_0} + (\nabla \cdot \mathbf{V})_{p_1}\right]$. If $\omega\,(1000\,\mathrm{hPa}) = 0$, then integrating upward gives

$$\begin{aligned}
\omega(850\,\mathrm{hPa}) &= 11.25 \times 10^{-2}\,\mathrm{Pa\,s^{-1}}\\
\omega(700\,\mathrm{hPa}) &= 18.00 \times 10^{-2}\,\mathrm{Pa\,s^{-1}}\\
\omega(500\,\mathrm{hPa}) &= 21.00 \times 10^{-2}\,\mathrm{Pa\,s^{-1}}\\
\omega(300\,\mathrm{hPa}) &= 15.00 \times 10^{-2}\,\mathrm{Pa\,s^{-1}}\\
\omega(100\,\mathrm{hPa}) &= -1.00 \times 10^{-2}\,\mathrm{Pa\,s^{-1}}
\end{aligned}$$

Now $w \approx -\left(\frac{RT}{g}\right)\left(\frac{\omega}{p}\right) = \left(-7.61 \times 10^3\right)\left(\frac{\omega}{p}\right)$. Thus, from the above expressions we estimate the vertical velocity at each level as

$$\begin{aligned}
w(850) &= -1.0\,\mathrm{cm\,s^{-1}}\\
w(700) &= -2.0\,\mathrm{cm\,s^{-1}}\\
w(500) &= -3.3\,\mathrm{cm\,s^{-1}}\\
w(300) &= -3.8\,\mathrm{cm\,s^{-1}}\\
w(100) &= +0.8\,\mathrm{cm\,s^{-1}}
\end{aligned}$$

3.23. Suppose that the lapse rate at the 850 hPa level is $4\,\mathrm{K\,km^{-1}}$. If the temperature at a given location is decreasing at a rate of $2\,\mathrm{K\,h^{-1}}$, the wind is westerly at $10\,\mathrm{m\,s^{-1}}$, and the temperature decreases toward the west at a rate of 5 K/100 km, compute the vertical velocity at the 850 hPa level using the adiabatic method.

Solution: From eq. (3.41) $\omega = S_p^{-1}\left[\partial T/\partial t + u\partial T/\partial x + v\partial T/\partial y\right]$. But $\omega \approx -\rho g w$, and $S_p = (\Gamma_d - \Gamma)/(\rho g)$. Thus, $w \approx -[\partial T/\partial t + u\partial T/\partial x + v\partial T/\partial y]/(\Gamma_d - \Gamma)$. $(\Gamma_d - \Gamma) = 9.8 \times 10^{-3} - 4 \times 10^{-3} = 5.8 \times 10^{-3}\,\mathrm{K\,km^{-1}}\,\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \left(\frac{-2}{3600}\right) + 10\left(\frac{5}{10^5}\right) + 0 = -5.56 \times 10^{-5}\,\mathrm{K\,s^{-1}}$, so that $w = -\left[\left(-5.56 \times 10^{-5}\right)/5.8 \times 10^{-3}\right] = 0.0096\,\mathrm{m\,s^{-1}}$, or 0.96 cm/s.