Vorticity

AOS660, Prof. McKinley

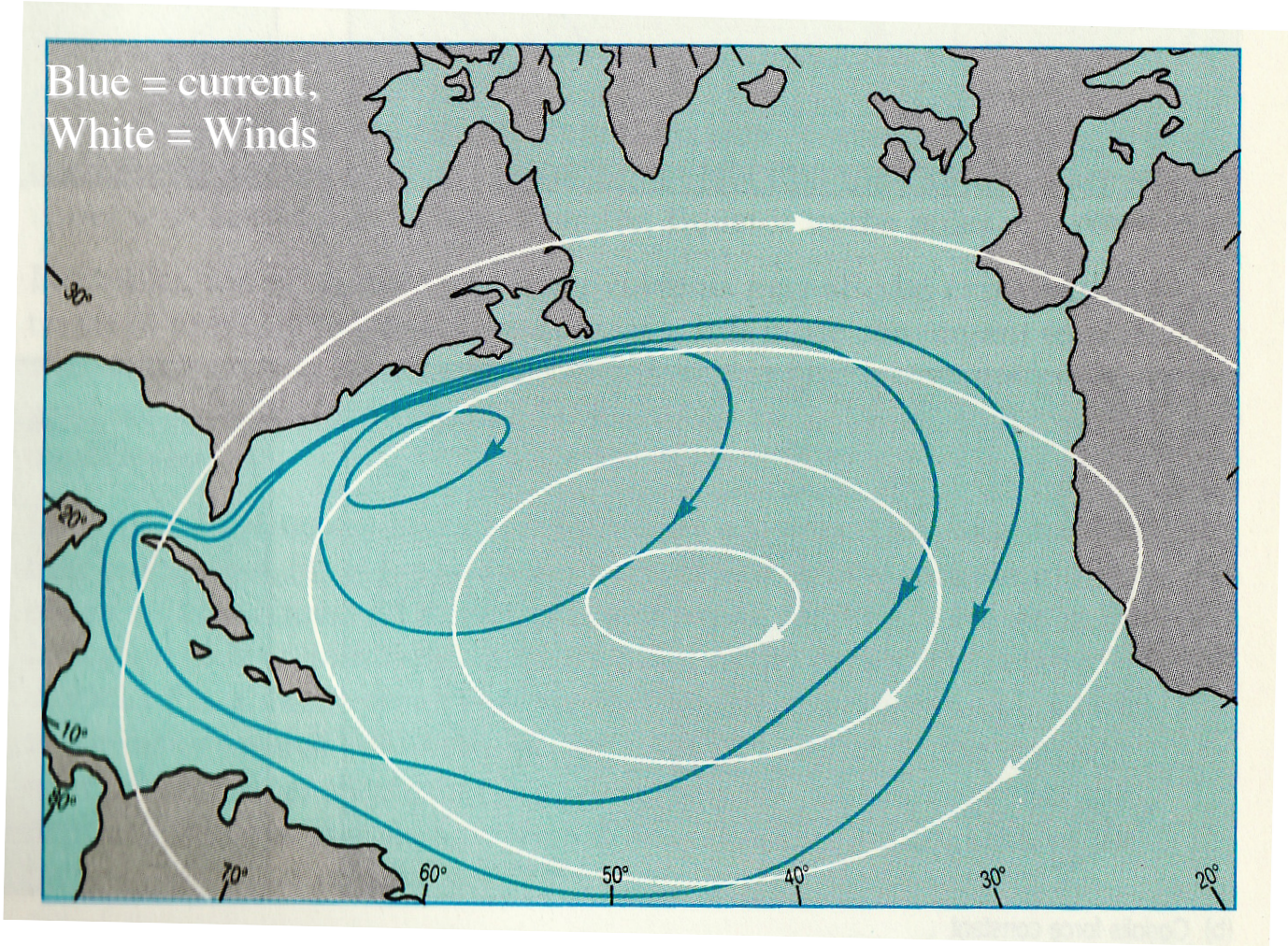
Fall 2013

- Vorticity
- Potential Vorticity

- Taylor Proudman Theorem
- Sverdrup flow

- Western Boundary Current

Wind drives the ocean, but why is there a Gulf Stream?



Open University, Ocean Circulation, Fig 4.11

Vorticity

Vorticity is the rotation of the fluid

Types of Vorticity

- Planetary Vorticity
- Relative vorticity
- Absolute vorticity = planetary + relative

Vorticity Equation

To derive: Allow acceleration terms,
so not assuming geostrophic

$$\frac{D}{Dt}(\zeta + f) + (\zeta + f)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0$$

Potential Vorticity

- "Shallow Water" equations
  - Barotropic
  - Allow height to vary

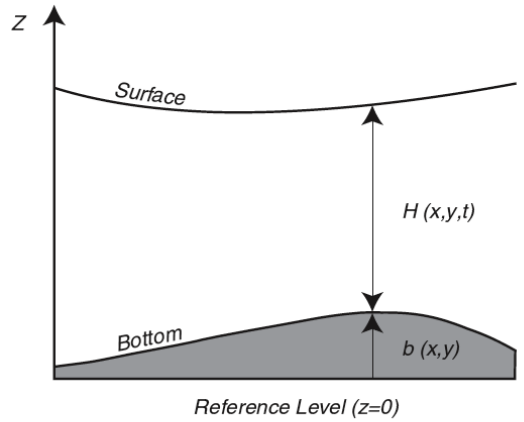


Figure 12.1 Sketch of fluid flow used for deriving conservation of potential vorticity. After Cushman-Roisin (1994: 55).

- Conservation of PV

$$\frac{D}{Dt}\left(\frac{\zeta + f}{H}\right) = 0$$

PV conservation example

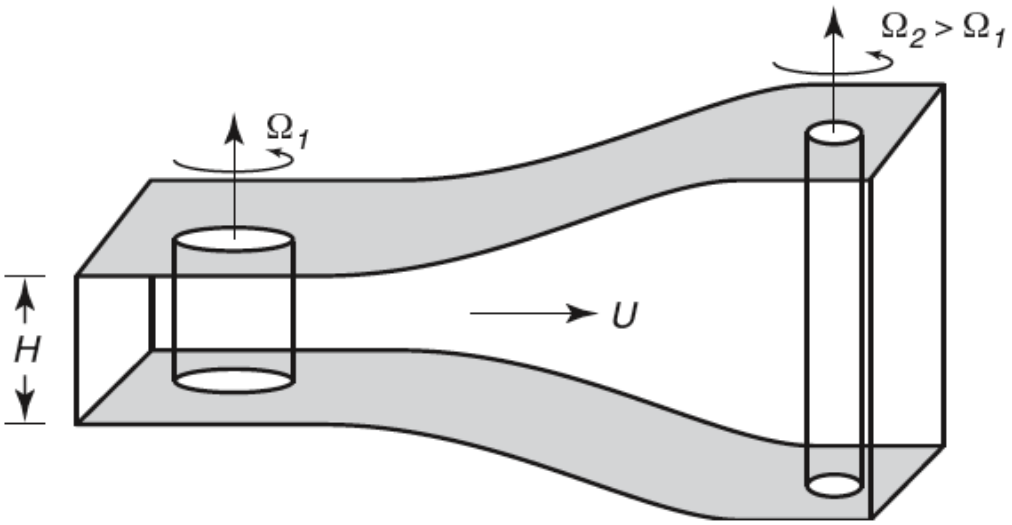


Figure 12.2 Sketch of the production of relative vorticity by the changes in the height of a fluid column. As the vertical fluid column moves from left to right, vertical stretching reduces the moment of inertia of the column, causing it to spin faster.

Exchange of relative and planetary vorticity with constant depth

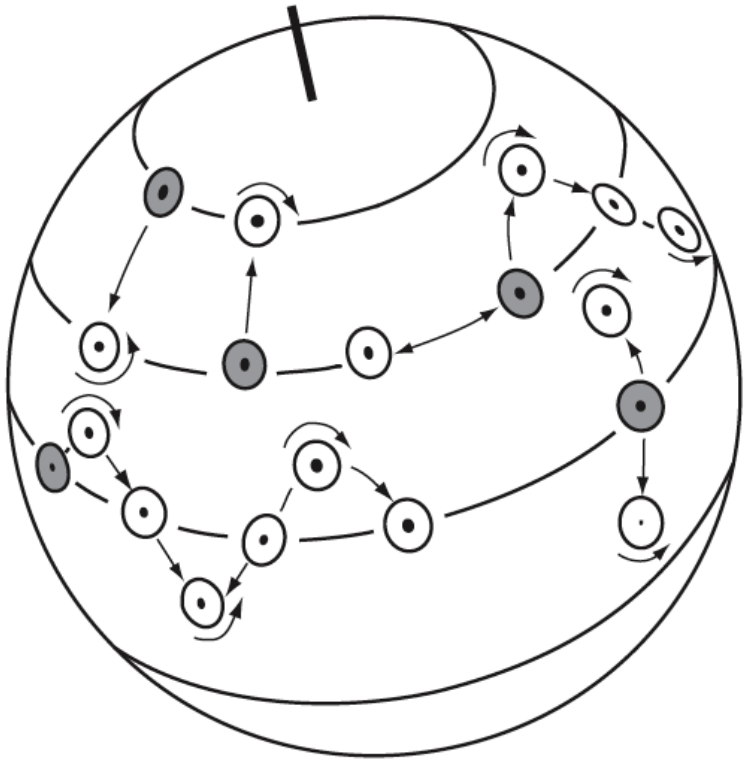


Figure 12.3 Angular momentum tends to be conserved as columns of water change latitude. This changes the relative vorticity of the columns. After von Arx (1962: 110).

Topographic Steering of Barotropic Flow

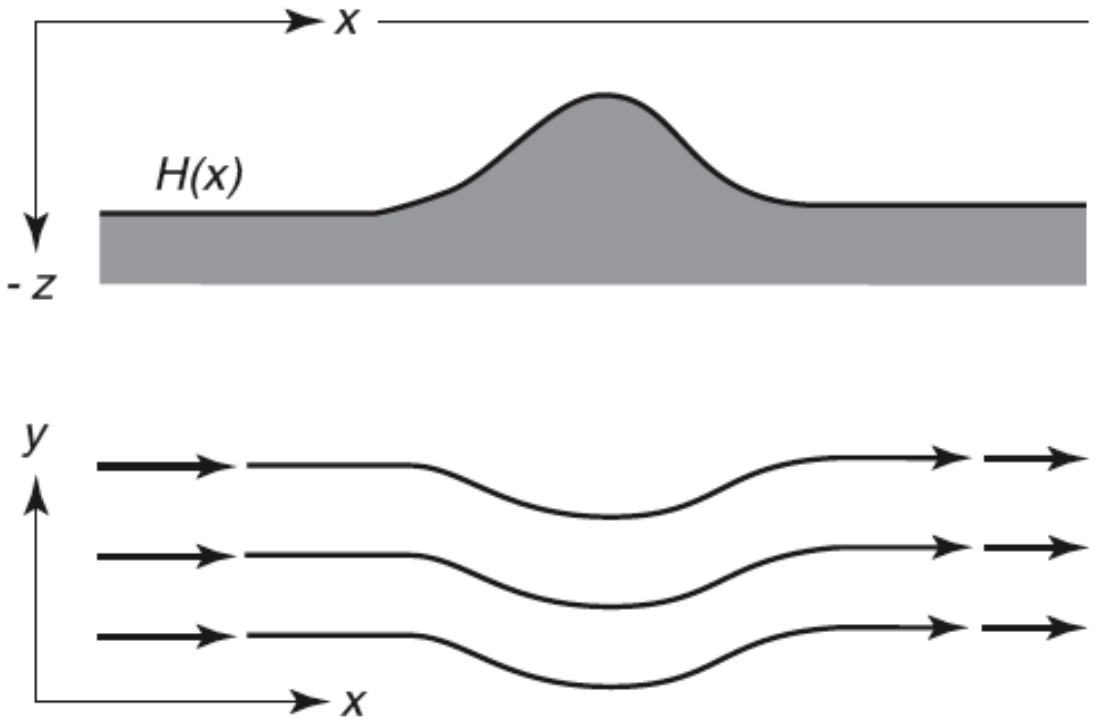


Figure 12.4 Barotropic flow over a sub-sea ridge is turned equatorward to conserve potential vorticity. After Dietrich et al. (1980: 333).

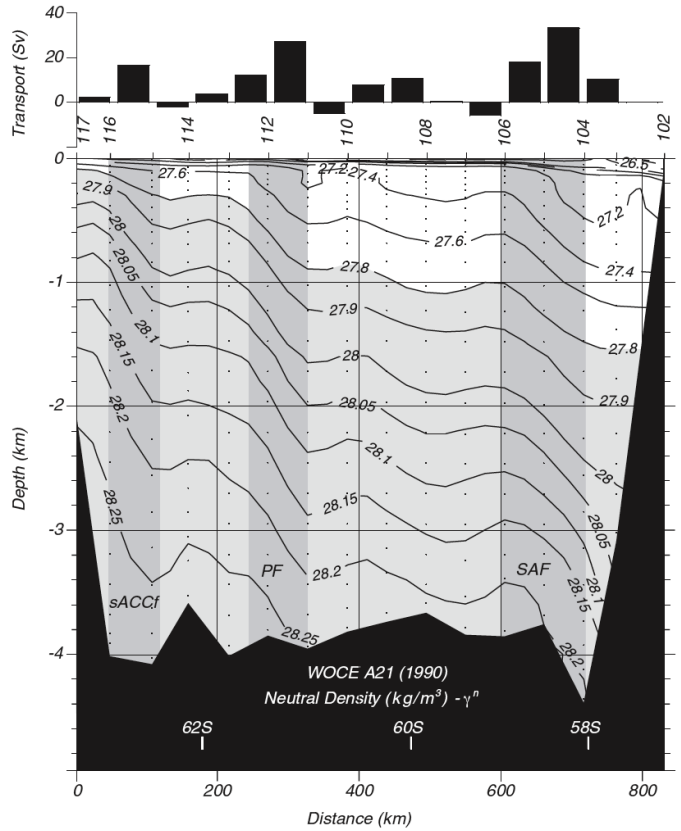
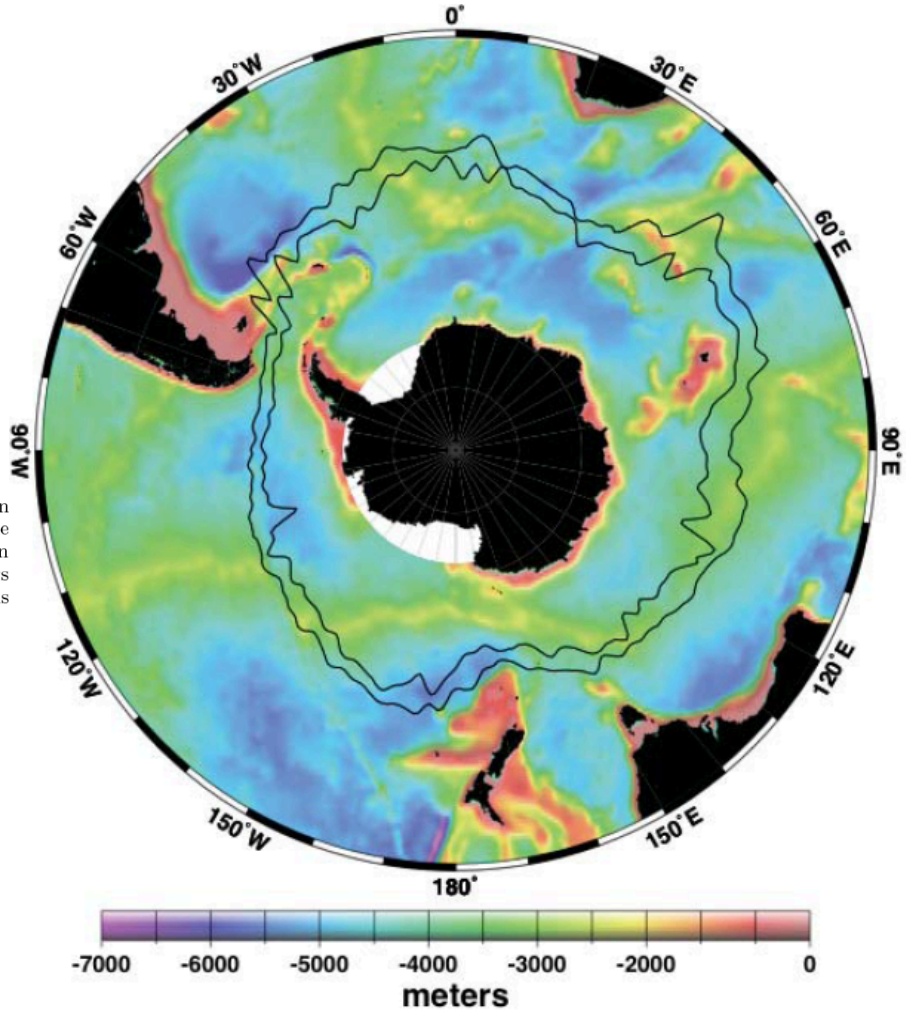


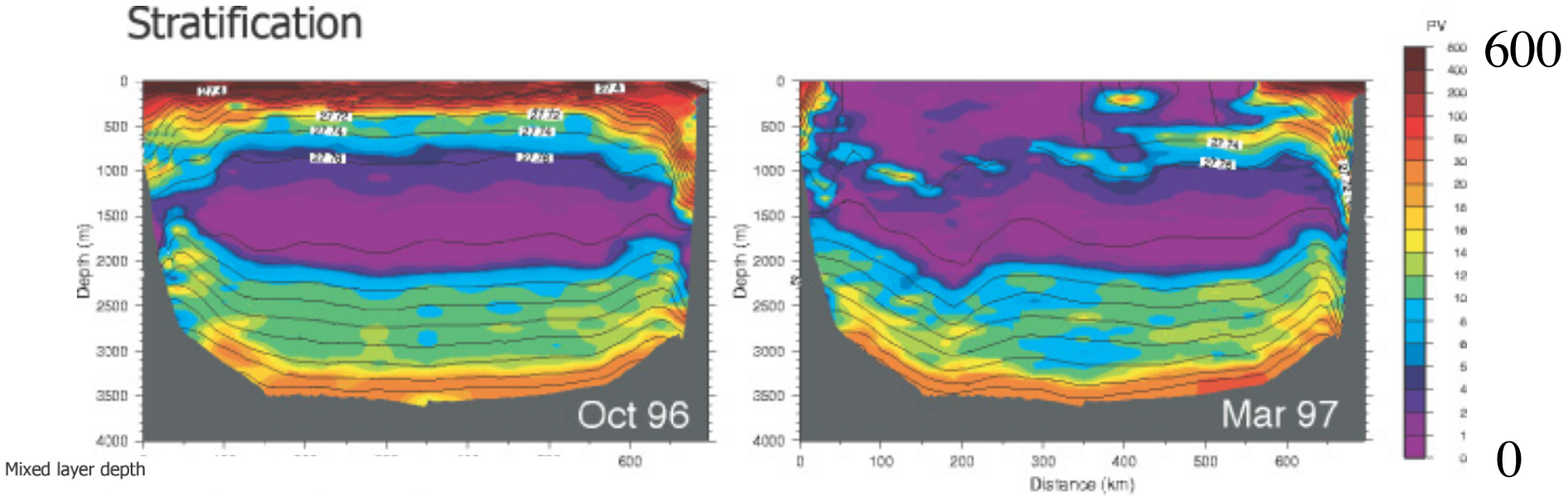
Figure 13.12 Cross section of neutral density across the Antarctic Circumpolar Current in the Drake Passage from the World Ocean Circulation Experiment section A21 in 1990. The current has three streams associated with the three fronts (dark shading): SF = Southern ACC Front, PF = Polar Front, and SAF = Subantarctic Front. Hydrographic station numbers are given at the top, and transports are relative to 3,000 dbar. Circumpolar deep water is indicated by light shading. Data from Alex Orsi, Texas A&M University.

Topographic Steering

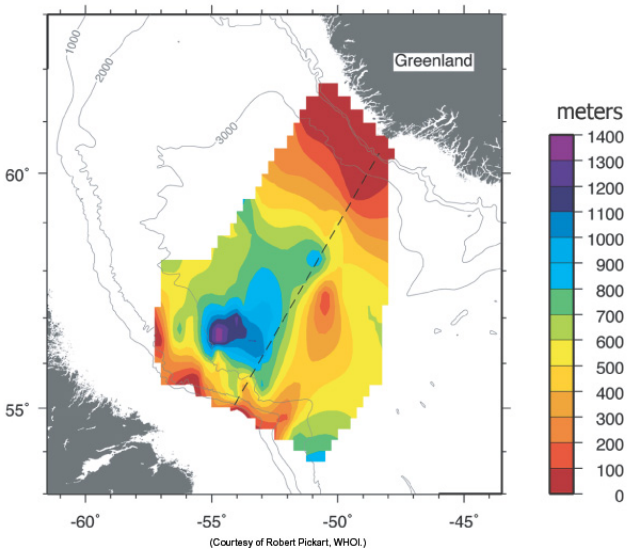


Stewart 2008
Gille et al. 2004

PV from Hydrography: Lab Sea



Mixed layer depth



Taylor Proudman

- geostrophic
- homogeneous
- f-plane

$$\frac{\partial u}{\partial z} = \frac{\partial v}{\partial z} = 0$$

$$\frac{\partial w}{\partial z} = 0 \text{ because geostrophic flow is non-divergent}$$

To derive Sverdrup flow, we relaxed the strict geostrophic assumption to allow vertical motion and df/dy to balance .. Can also consider this as Taylor Proudman on a Sphere

$$\beta v_g = f \frac{\partial w_e}{\partial z}$$

$$\text{Integrated form: } \beta V = f w_e$$

Taylor Proudman on a sphere

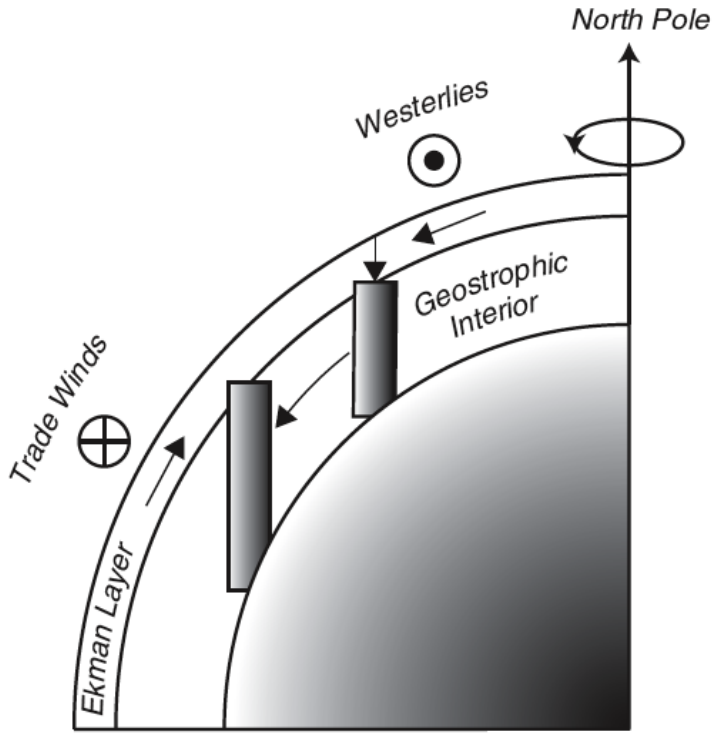
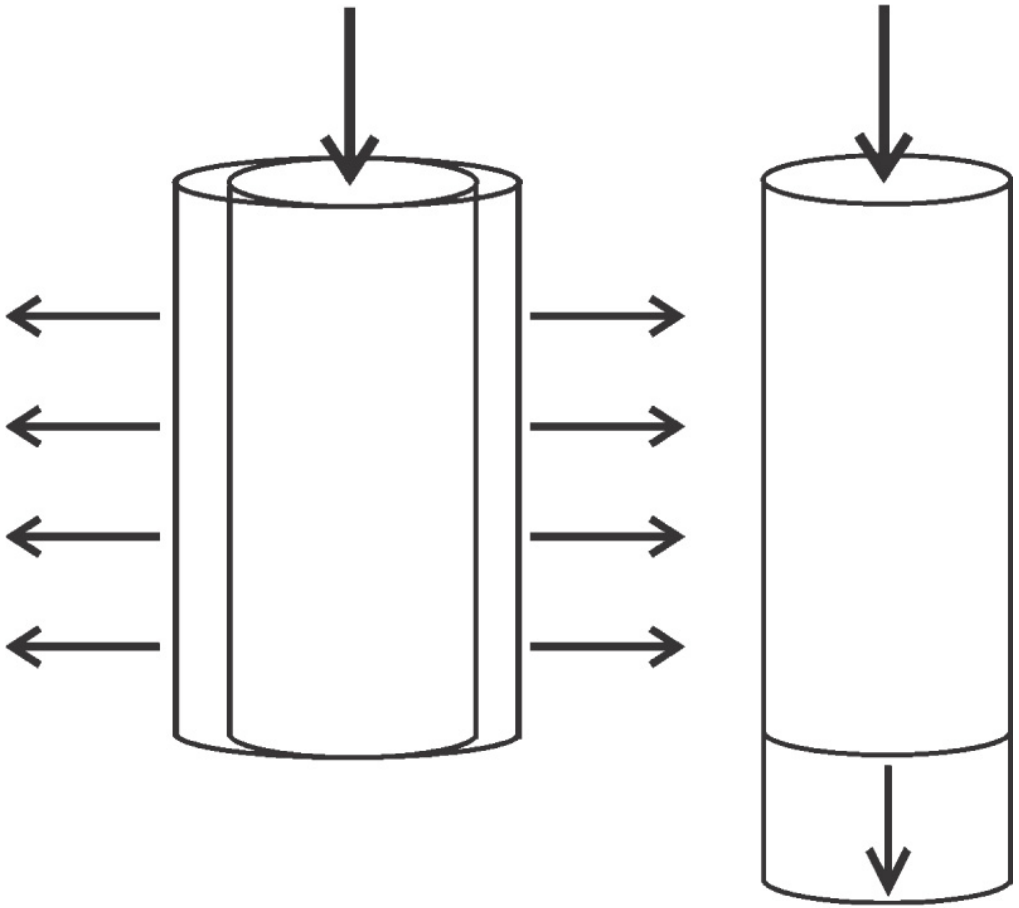
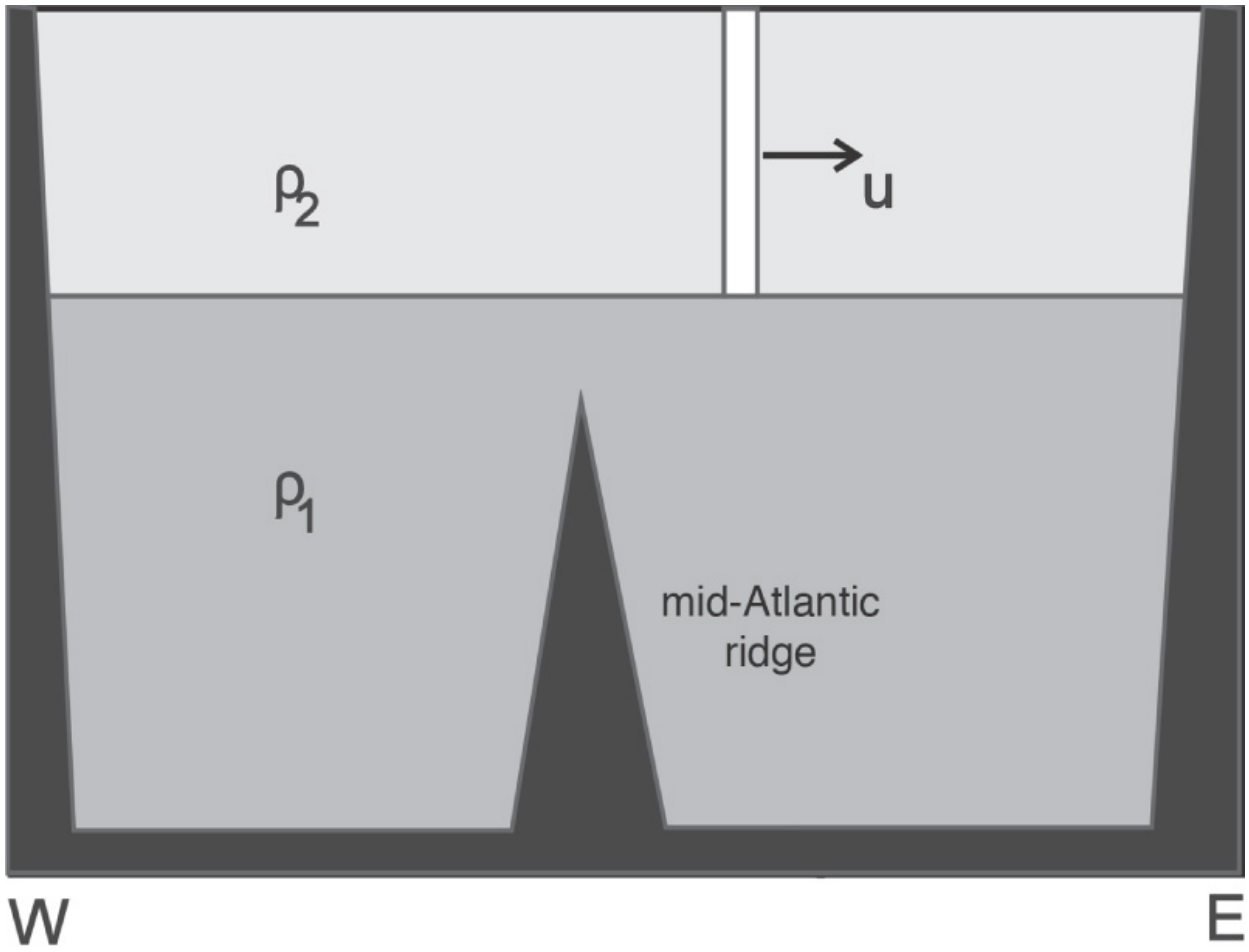


Figure 12.6 Ekman pumping that produces a downward velocity at the base of the Ekman layer forces the fluid in the interior of the ocean to move southward. See text for why this happens. After Niiler (1987).

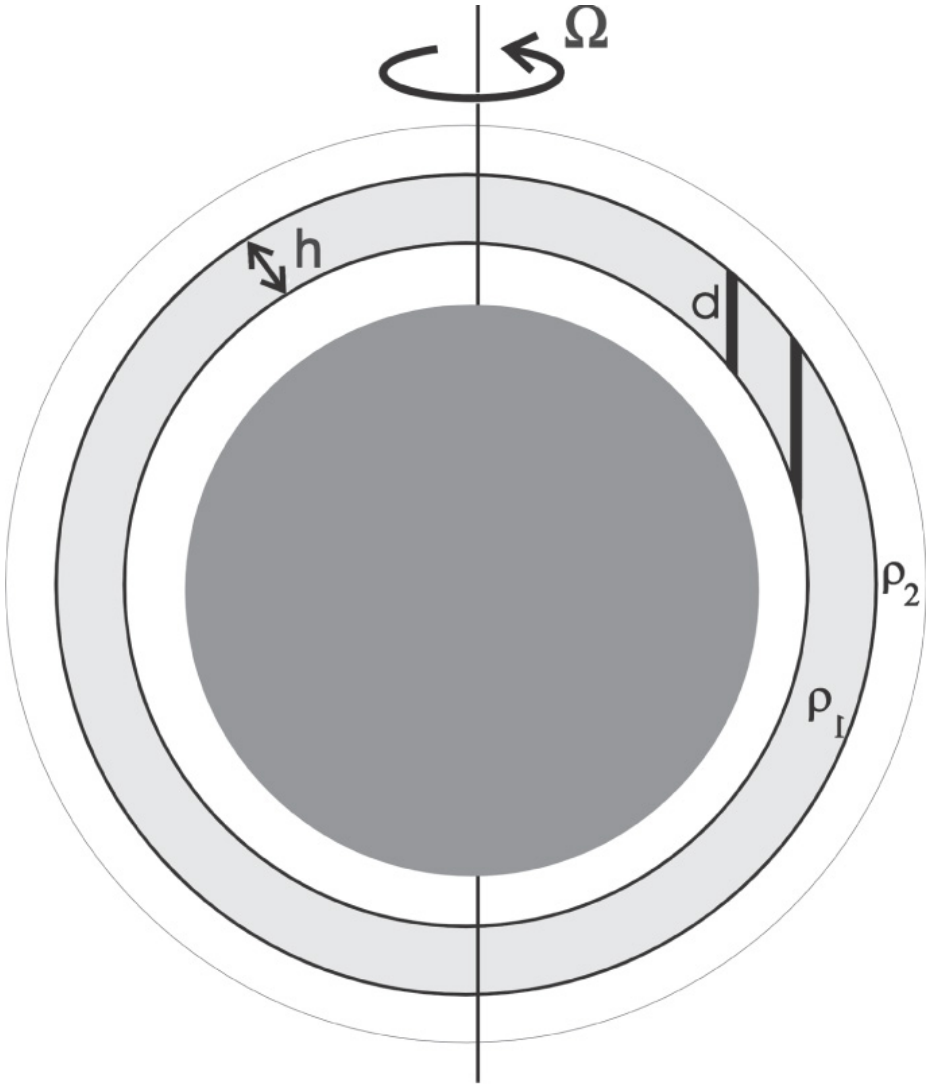
Why not decrease ζ to accommodate w_e?



Ocean is actually stratified



T-P in layered ocean

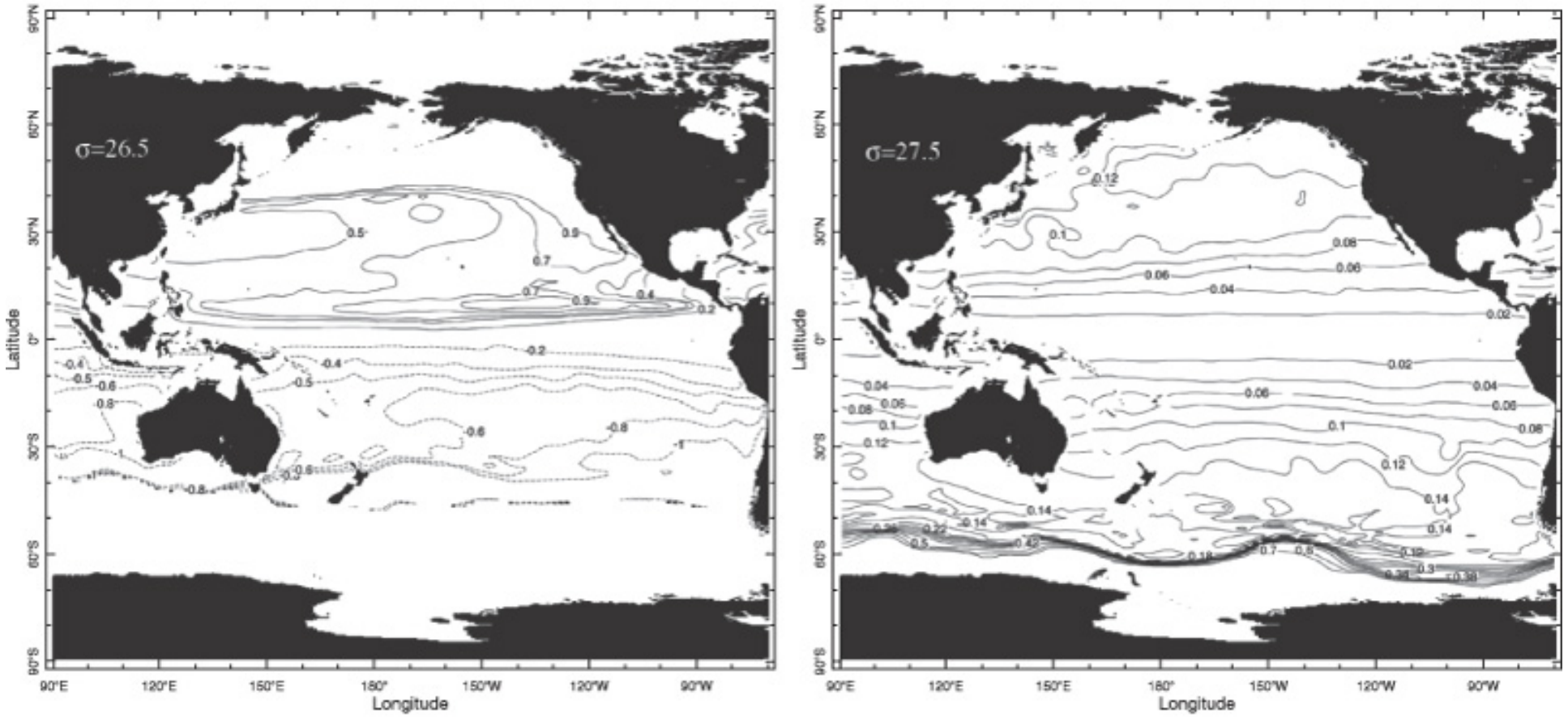


From data

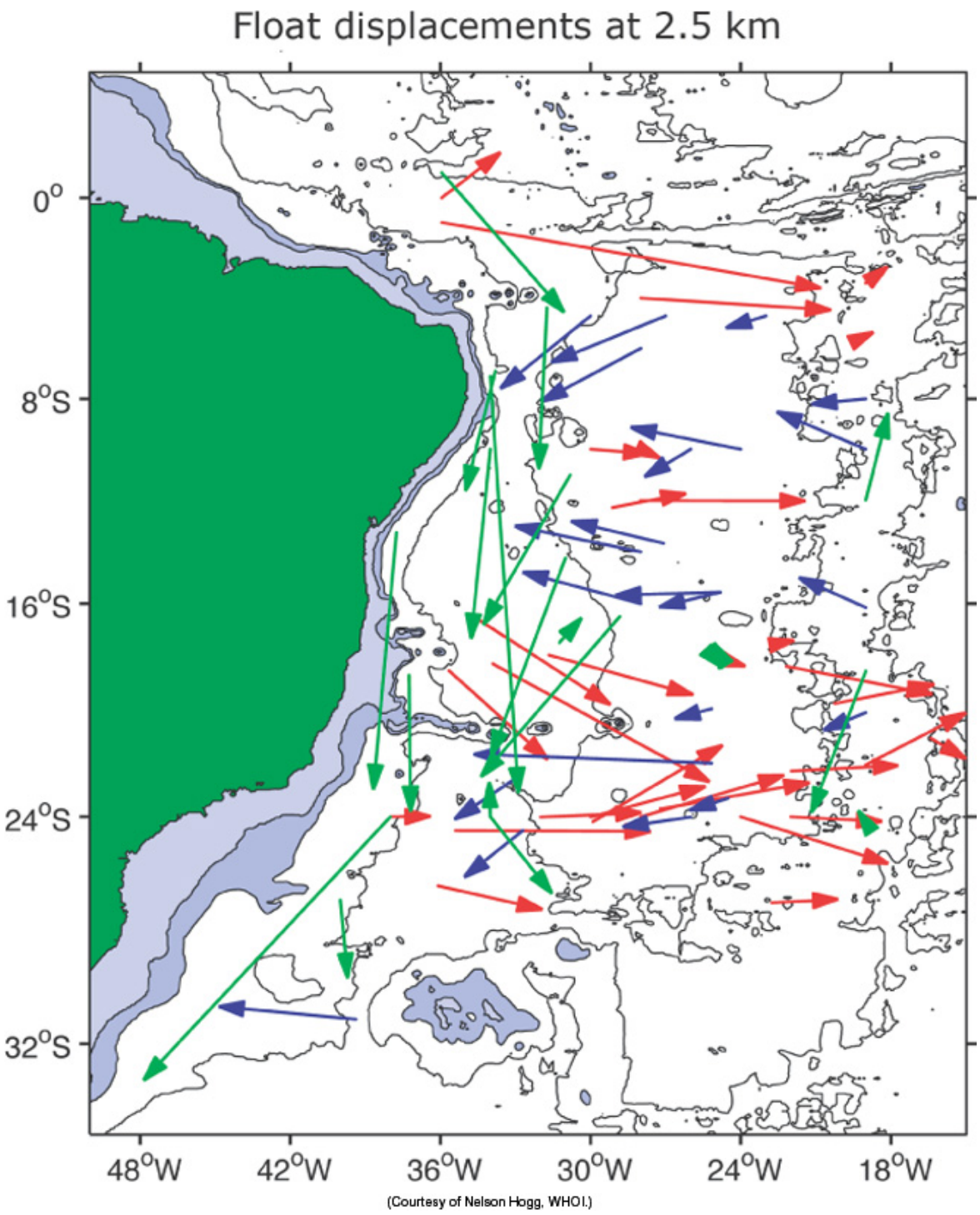
Sigma = 26.5

f/h on σ

Sigma = 27.5



2.5km Float observations indicate primarily zonal flow at depth



Why a Western Boundary Current (WBC)?

- Heuristic Argument – Flow is already "going that way" -- no reason to turn back on itself

- Vorticity Argument addresses not only why we have a WBC, but also the overall vorticity budget of the gyre

Vorticity explanation for Western Boundary Currents

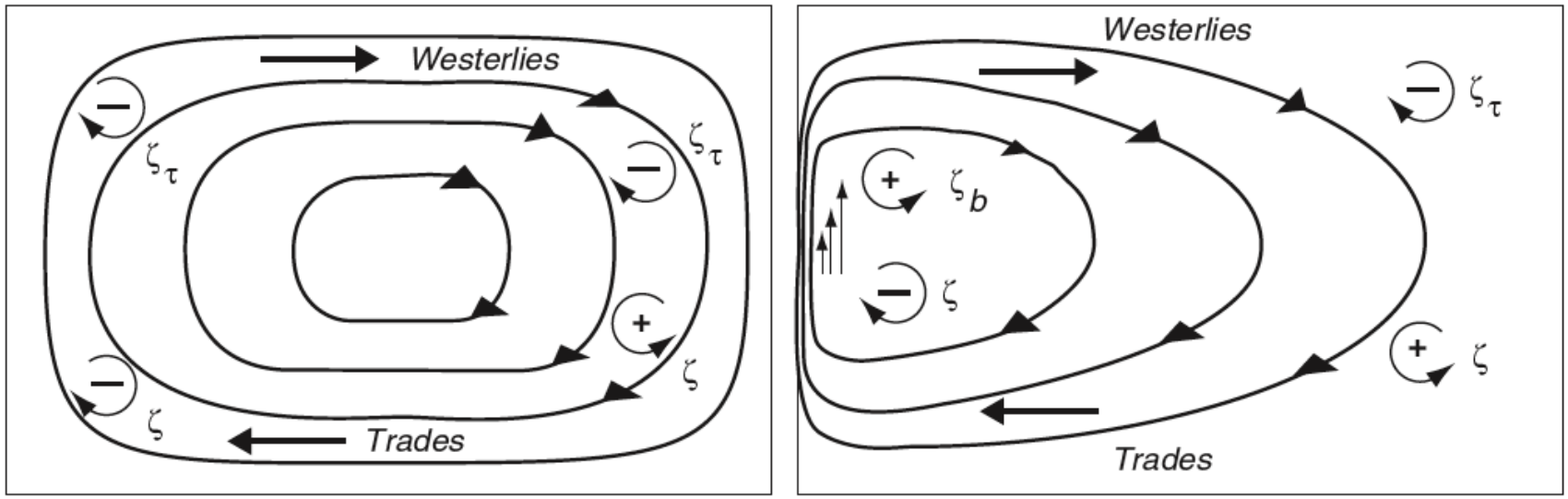


Figure 12.5 The balance of potential vorticity can clarify why western boundary currents are necessary. **Left:** Vorticity input by the wind ζ_τ balances the change in relative vorticity ζ in the east as the flow moves southward and f decreases. The two do not balance in the west where ζ must decrease as the flow moves northward and f increases. **Right:** Vorticity in the west is balanced by relative vorticity ζ_b generated by shear in the western boundary current.