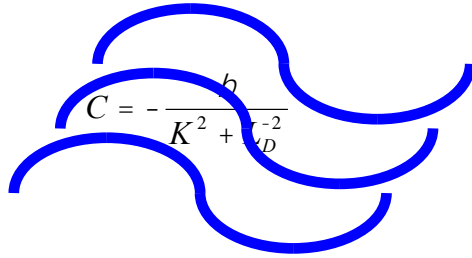*AOS 611*, **Spring, 2013**

**Geophysical Fluid Dynamics II:**

**Large Scale Dynamics.**

**Prof. Z. Liu**

**Department of Atmospheric and Oceanic Sciences**

**University of Wisconsin - Madison**

$$C = -\frac{b}{K^2 + L_D^{-2}}$$

# AOS 611: Syllabus, Spring, 2013

# Geophysical Fluid Dynamics II:Large Scale Dynamics[1]

## Prof. Z. Liu, Dept. Atmos. Oceanic Sci., UW-Madison

## Chapter 0. Introduction

## Part I: Dynamics of Shallow Water System

## Chapter 1: Basics  (3 weeks)

Sec.1.0: Introduction
Sec.1.1: Basic equations,
Sec.1.2: Conservation laws *
Sec.1.3: Circulation, vorticity and Kelvin's Theorem *
Sec.1.4: Potential vorticity conservation
Sec.1.5: Shallow water waves on a f-plane
Sec.1.6: Geostrophic adjustment

## Chapter 2: Shallow Water Rossby Wave Dynamics (3 weeks)

Sec.2.1: Quasi-geostrophic equation
Sec.2.2: Rossby waves
Sec.2.3: Group velocity and energy propagation
Sec.2.4: Reflection and normal modes
Sec.2.5: Forced waves
Sec.2.6: Non-plane waves*

## Chapter 3: Forced Circulation (1.5 week)

Sec.3.1: Atmospheric circulation
Sec.3.2: Ekman dynamics
Sec.3.3: Sverdrup flow
Sec.3.4: Rossby wave and ocean circulatioin

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[1] Most sections requires both understanding and derivation of the equations. Sections with * requires understanding only. Sections with ! are not required.

**<u>Part II: Dynamics of Stratified Flow</u>**


**Chapter 4: Basics of Stratified Fluid** (1.5 weeks)

Sec.4.1: Basic equations
Sec.4.2: Vorticity equation
Sec.4.3: Ertel potential vorticity *

**Chapter 5: Rossby Wave Dynamics** (2 weeks)

Sec.5.1: Quasi-geostrophic equation for stratified flow *
Sec.5.2: Rossby waves in stratified fluid
Sec.5.3: Vertical normal modes
Sec.5.4: The Elliassan-Palm theorem *


**Chapter 6: Instability Theory**  (2 weeks)

Sec.6.1: Instability problem
Sec.6.2: Baroclinic instability in a two-layer QG model
Sec.6.3: Energetics
Sec.6.2: Charney-Stearn theorem *
Sec.6.4: The Eady problem. *
Sec.6.6: Barotropic instability *


**Grading**:

30 % homework, 30% mid-term exam,  40% final exam

**References:**

1: Pedlosky, J. Geophysical Fluid Dynamics (2nd ed, 1987)**,** *Springer-Verlag***.**

2:  Gill, A. E., Atmosphere-Ocean Dynamics, 1981, *Academic Press*.

3: Holton, J. R., An Introduction to Dynamic Meteorology (3rd ed), *Academic Press*.

## Chapter 0. Introduction

Why do we study dynamics? The ultimate forcing of the atmospheric and oceanic circulation is the solar radiation. The ocean and atmosphere systems adjust themselves such that the system is in a dynamic equilibrium.

### 1. Radiative Equilibrium

At the first order, the equilibrium is the so called radiative equilibrium. At this state, the incoming solar radiation warms the surface of the earth. The surface is cooled by the outgoing long wave radiation into the space. Since the atmosphere absorbs mainly long wave radiation from the surface, the atmosphere is heated from below. This first picture of the atmospheric equilibrium, although simple, can explain some essential features of the observed atmosphere stratification. For example, air temperature decreases away from the surface. The surface temperature is about $200^{o}K$ to $300^{o}K$ from the high to low latitudes.

This radiative equilibrium, however, still differs substantially from the observation in several important aspects. First, the vertical change of air temperature is too large: 150 K from surface to 10 Km as opposed to about 70K in the observation; (Fig.0.1). Second, the latitudinal temperature gradient is also two large, about 100 K from pole to equator, as opposed to about 50 K in the observation (Fig.0.2). Therefore, the pure radiative equilibrium atmosphere has temperature gradients about twice that of the observation in both the vertical and latitudinal directions. Dynamic processes transport heat (Fig.0.3) to reduce the mean temperature gradients and therefore play important roles in maintaining the mean climatology.

In comparison with the atmosphere, the dynamic processes are even more important in the ocean. This is because radiation only penetrates the surface ocean water by about 10 meters or so. The stratification of the entire subsurface ocean (Fig.0.4), therefore, at the first order, is determined by oceanic dynamic transports.
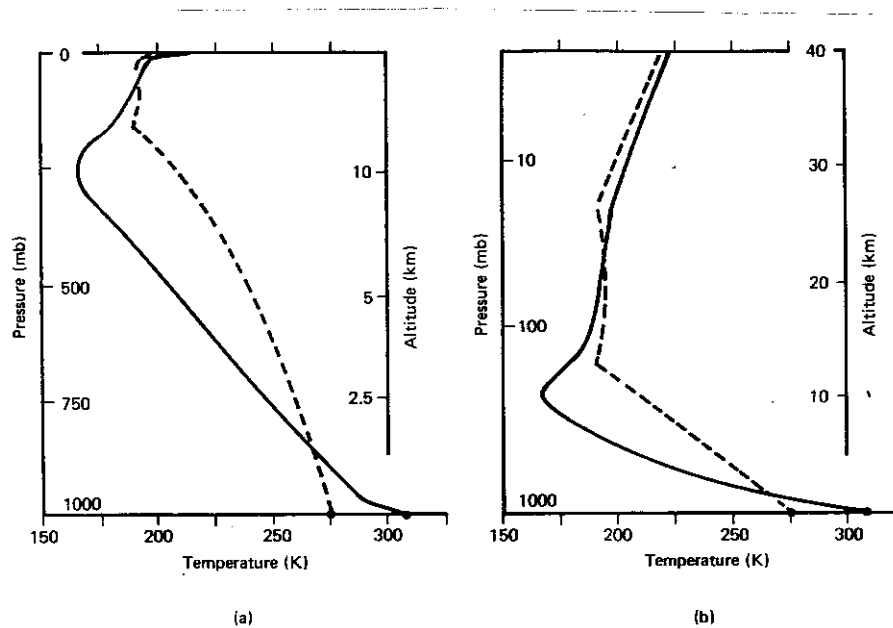
Fig. 1.4.   The radiative equilibrium solution (solid line) corresponding to the observed distribution of atmospheric absorbers at 35°N in April, the observed annual average insolation for the whole atmosphere, and no clouds. The dashed line shows the effect of convective adjustment to a constant lapse rate of 6.5 K km$^{-1}$. In (a) the curves are drawn with a scale linear in pressure, i.e., equal intervals correspond to equal masses of atmosphere. In (b) the scale is linear in altitude. [From Manabe and Strickler (1964, Fig. 4).]

# Figure 0.1: Radiative and radiative convective equilibrium in the atmosphere
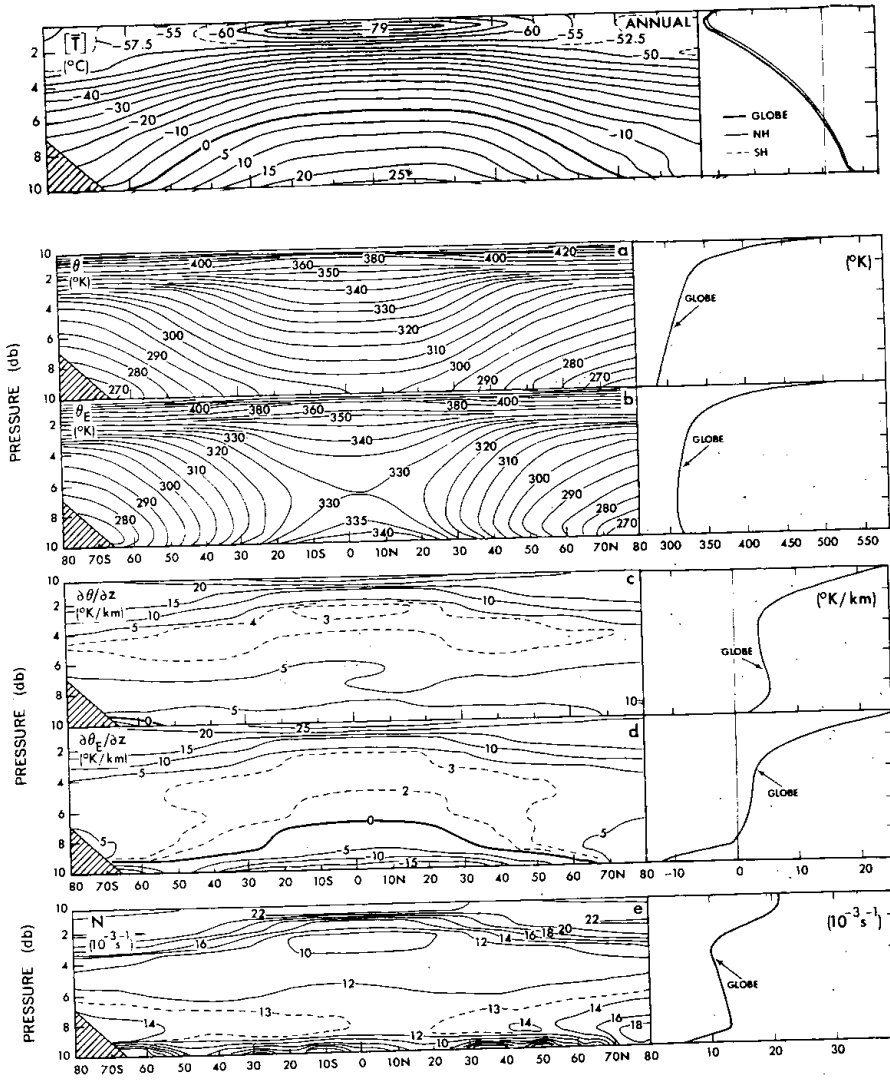
**FIGURE 7.6.** Zonal-mean cross sections of the potential temperature (a) in K, equivalent potential temperature (b) in K, vertical gradient of potential temperature (c) in K/km, vertical gradient of equivalent potential temperature (d) in K/km, and Brunt–Väisälä frequency in $10^{-3}$ rad s$^{-1}$ for annual-mean conditions. Vertical profiles of the global mean values are shown on the right.

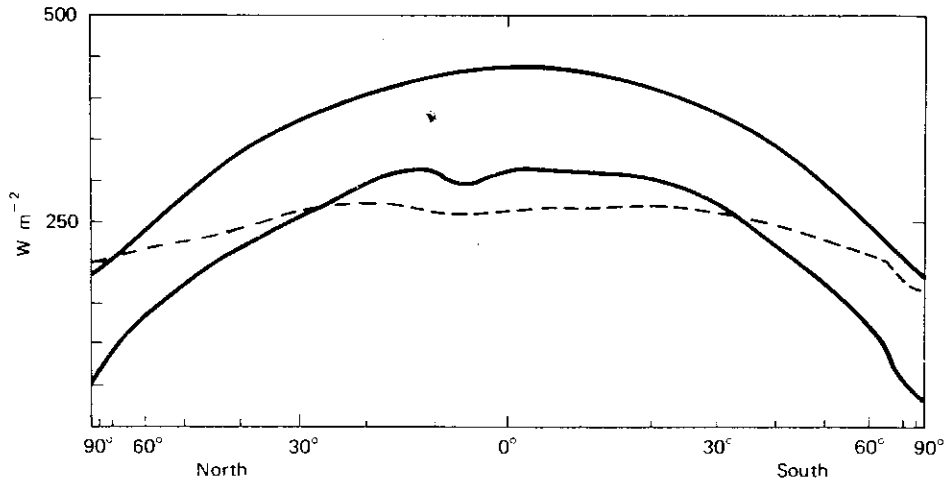## Figure 0.3: Zonal mean potential temperature of the atmosphere

**Fig. 1.1.** The radiation balance of the earth. The upper solid curve shows the average flux of solar energy reaching the outer atmosphere. The lower solid curve shows the average amount of solar energy absorbed; the dashed line shows the average amount of outgoing radiation. The lower curves are average values from satellite measurements between June 1974 and February 1978, and are taken from Volume 2 of Winston et al. (1979). Values are in watts per square meter. The horizontal scale is such that the spacing between latitudes is proportional to the area of the earth's surface between them, i.e., is linear in the sine of the latitude.
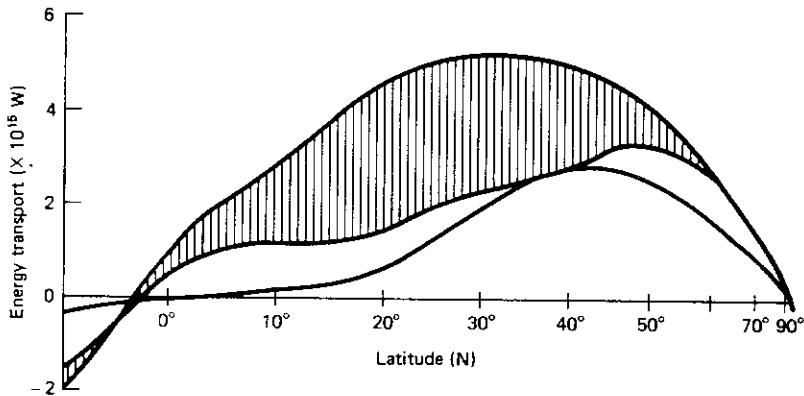


**Fig. 1.8.** The northward transport of energy (in units of petawatt = $10^{15}$ W) as a function of latitude. The outer curve is the net transport deduced from radiation measurements. The white area is the part transported by the atmosphere and the shaded area the part transported by the ocean. The lower curve denotes the part of the atmospheric transport due to transient eddies and is the mean of the monthly values from Oort (1971, Table 3). The horizontal scale is such that the spacing between latitudes is proportional to the area of the earth's surface between them, i.e., is linear in the sine of the latitude. [From Vonder Haar and Oort (1973).]

# Figure 0.3: Latitudinal distribution of radiation forcing and heat transports in the atmosphere and ocean
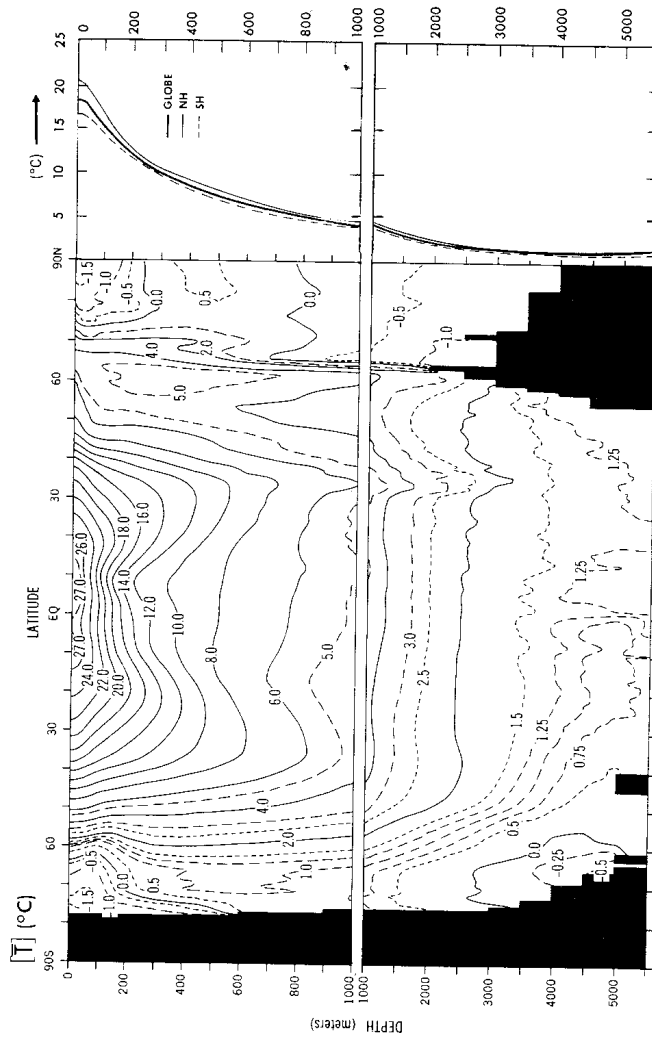
FIGURE 8.4. Zonal-mean cross section of temperature for the top ocean layer 0–1000 m and the layer below 1000 m in °C for annual-mean conditions. Vertical profiles of the hemispheric and global mean temperatures are shown on the right (after Levitus, 1982).

**Figure 0.4: Zonal mean temperature of the ocean**

## 2. Radiative-convective equilibrium (vertical)

In the atmosphere, at the next order is the covnective. The cold air is too heavy in the upper atmosphere such that the air column become convectively unstable. Convection reduces the vertical temperature gradient significantly to a neutral stable stratification – radiative-convective equilibrium (about $6.5^oK/1km$ in the moist saturate atmosphere and $10^oK/1km$ in the dry atmosphere) (Fig.0.1)
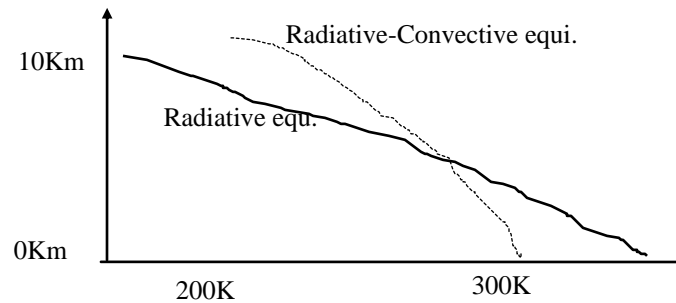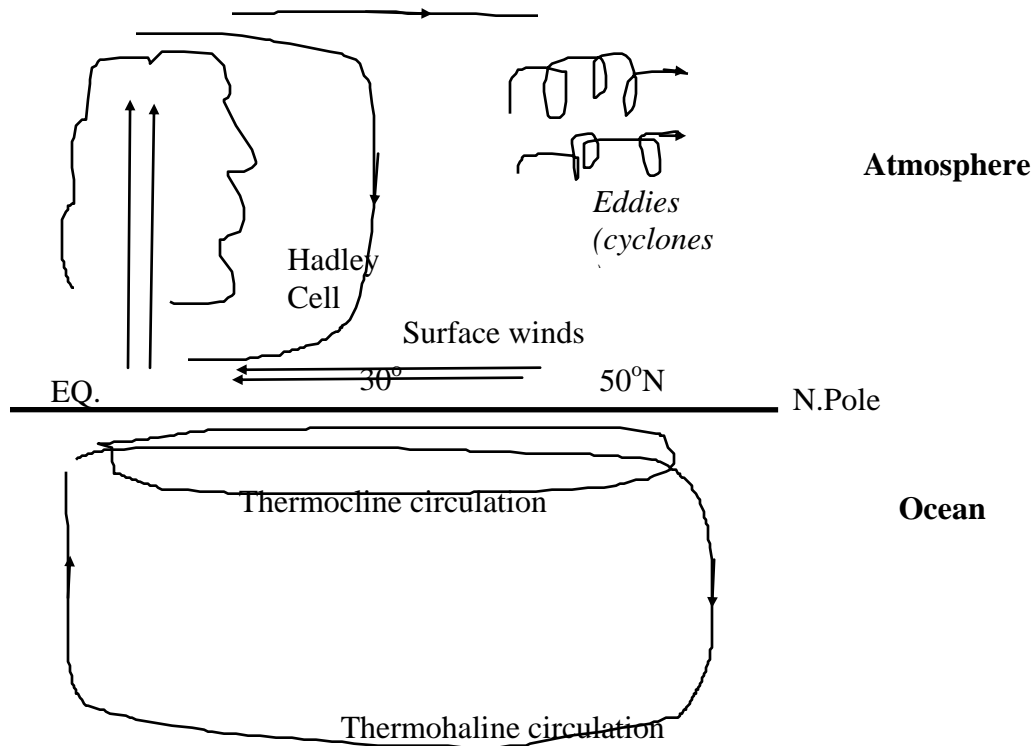


Convection also plays important role in setting the deep stratification at the high latitude ocean, and in turn the deep circulation and stratification. Different from the atmosphere, however, oceanic convection alone adjusts the water column in the subsurface ocean to convective equilibrium, rather than convective-radiative equilibrium.

## 3. Transport: dynamics (horizontal)

Even after vertical dynamic process, we are still left with important questions. In the atmosphere, the latitudinal gradient of temperature in the atmosphere is still too large. Even worse is the ocean. Neither the horizontal nor the vertical temperature distribution can be explained by the local equilibrium.

The global adjustment will be accomplished by the horizontal transport in the atmosphere and ocean – the so called dynamic processes. The heat transport is partitioned about equally between the atmosphere and ocean (Fig.0.3).Like the vertical convective process that is trying to bring the system to gravitational stability, the horizontal transport of heat eventually bring the climate system to some sort of equilibrium. As a result, the horizontal temperature gradient will also be reduced greatly.

**Atmosphere**

*Eddies (cyclones*

Hadley Cell

Surface winds

EQ.          30°          50°N          N.Pole

Thermocline circulation          **Ocean**

Thermohaline circulation

In general, the atmospheric process involves the Hadley Cell (Fig.0.5) in the low latitude and storm track activity in the midlatitude. The oceanic process involves shallow thermocline processes and the thermohaline process (Fig.0.6). The former is wind-driven while the latter is buoyancy driven.
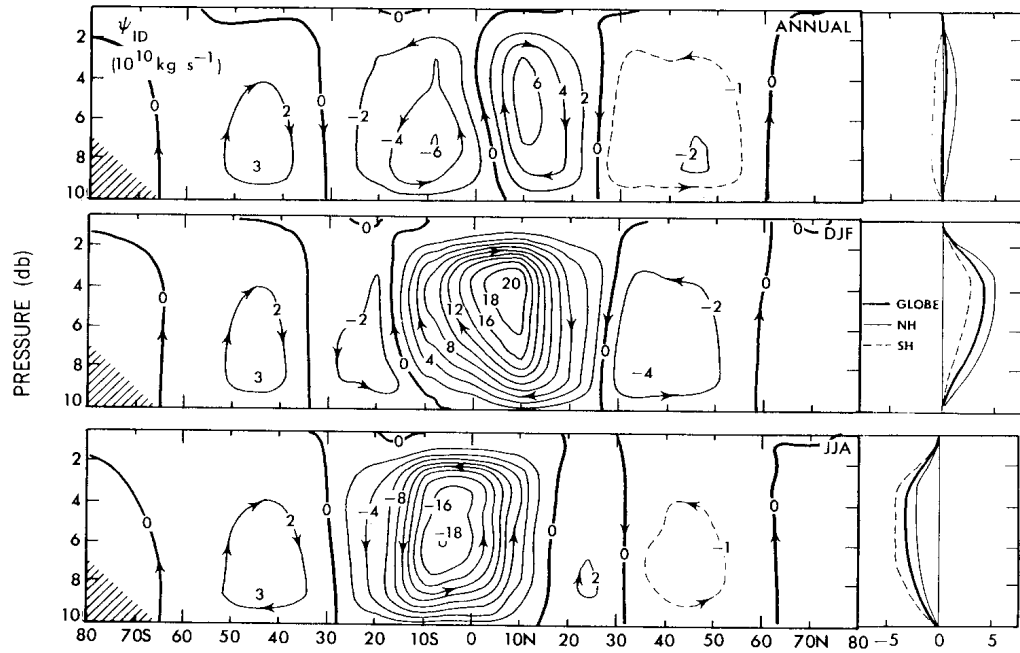
**FIGURE 7.19.** Zonal-mean cross sections of the mass stream function in $10^{10}$ kg s$^{-1}$ for annual, DJF, and JJA mean conditions. Vertical profiles of the hemispheric and global mean values are shown on the right.

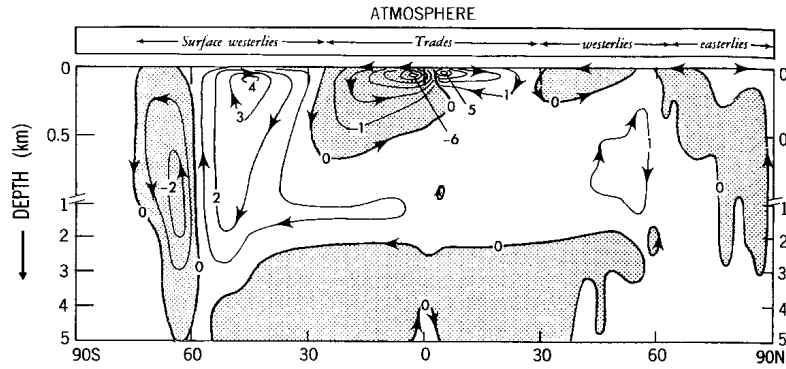**Fig.0.5: Zonal mean circulation in the atmosphere**

**FIGURE 8.20.** Zonal-mean cross section of the annual-mean flow of mass in $10^{10}$ kg s$^{-1}$ for a ocean model (adapted from Bryan and Lewis, 1979).

## Figure 0.6: Zonal mean circulation in the ocean

## 4. Comparison of atmospheric and oceanic dynamics

The dynamics in the atmosphere and ocean have many similarities and differences.  We should try to keep in mind of both systems. This will give us a deeper understanding of both systems.

a) Similarities:

Both systems are affected by: i) rotation, ii) stratification, iii) global scale processes. It is these similarities that makes the GFD here in principle apply to both systems, and makes the GFD different from classical fluid dynamicds.

b) Differences:

|  | **Atmosphere** | **Ocean** |
|---|---|---|
| Forcing | pure buoyancy driven | wind and buoyancy driven |
| Horizontal Domain | no horizontal boundary | with horizontal boundary |
| Vertical | No upper boundary | Bounded vertically |
| Weight, speed | light and fast | heavy and slow |
| In addition to temperature | water vapor | salinity |
| radiation | Radiative critical | Dynamics critical |
| stratification | strong | weak |
| compressibility | compressible | incompressible |
|  | …. | …. |