Dynamics
The basic equations

AOS660, Prof. McKinley, Fall 2013

- Key conservation equations
- Total derivative
- Navier-Stokes = conservation of momentum
- Continuity
- Energy Conservation
- Scaling
- Geostrophy
- Thermal Wind in the ocean

Key equations: Newtonian Mechanics

- Conservation of Momentum = Navier-Stokes
- Conservation of Mass = Continuity
- Conservation of Energy

Apply In

- Cartesian Coordinate System
  - x,y,z
  - positive to east, north, up

- Frame
  - Absolute or Inertial
  - Rotating or Non-intertial
  - Rotating Sphere

Apply to a moving fluid parcel

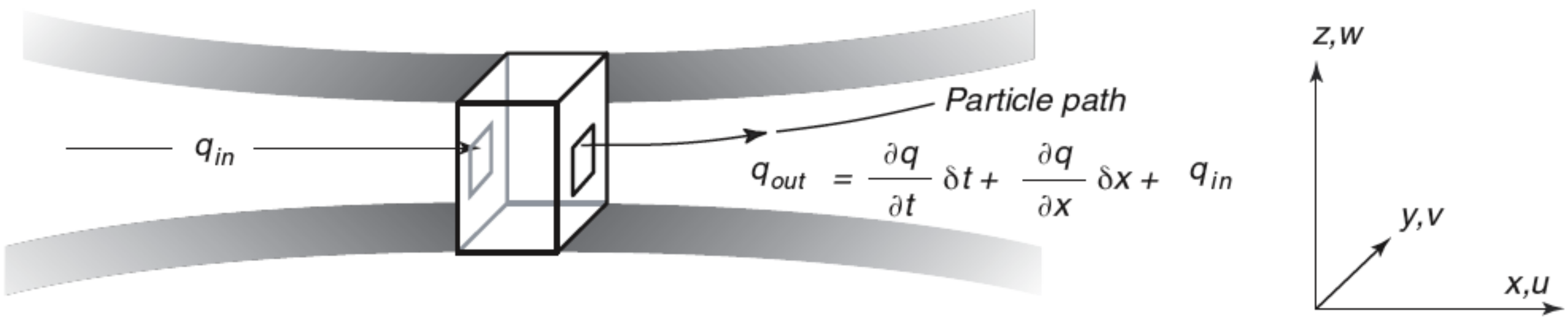


Figure 7.2 Sketch of flow used for deriving the total derivative.

Total Derivative "Lagrangian motion"

Forces in Absolute Frame

Forces in Absolute Frame

- Gravity (vertical)
  - Buoyancy is gravity acting on difference in density
- Pressure gradient (horizontal, vertical)
- Friction (horizontal, vertical)

- Lead to accelerations (horizontal, vertical)

Force Balance in Absolute Frame

$$\frac{D\vec{u}}{Dt} = -\frac{1}{\rho}\vec{\nabla} p - \vec{g} + \vec{F_r}$$

Forces in Rotating Frame

- Real
  - Gravity (vertical)
    - Buoyancy is gravity acting on difference in density
  - Pressure gradient (horizontal, vertical)
  - Friction (horizontal, vertical)

- Apparent
  - Coriolis (horizontal)

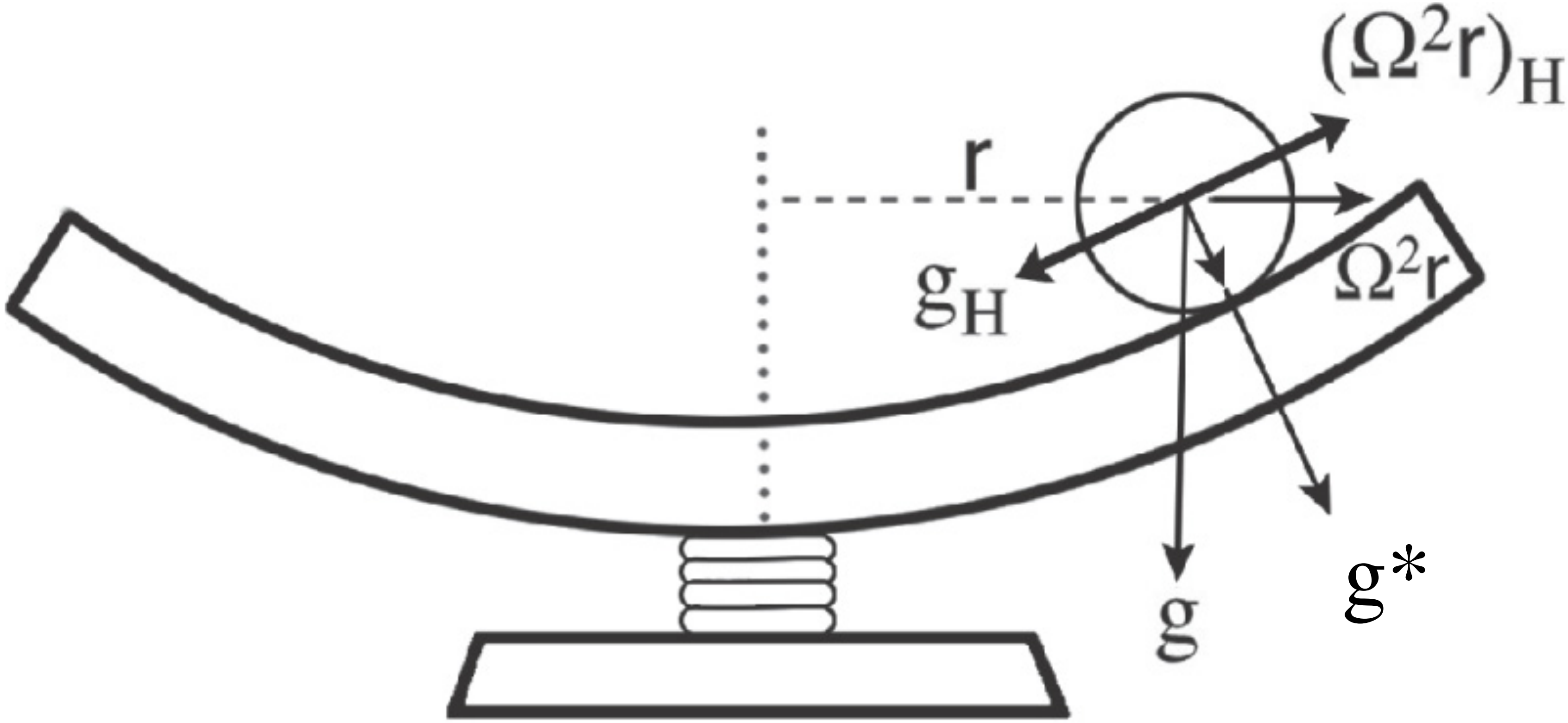
    http://en.wikipedia.org/wiki/File:Corioliskraftanimation.gif

Coriolis Force

$$CF = -2\vec{\Omega} \times \vec{u}$$

$\Omega = 7.292 \times 10^{-5}$ 1/s

AND incorporate centrifugal into g*



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Navier Stokes puts these forces into a balance equation for conservation of momentum acting on a Lagrangian parcel in a rotating frame

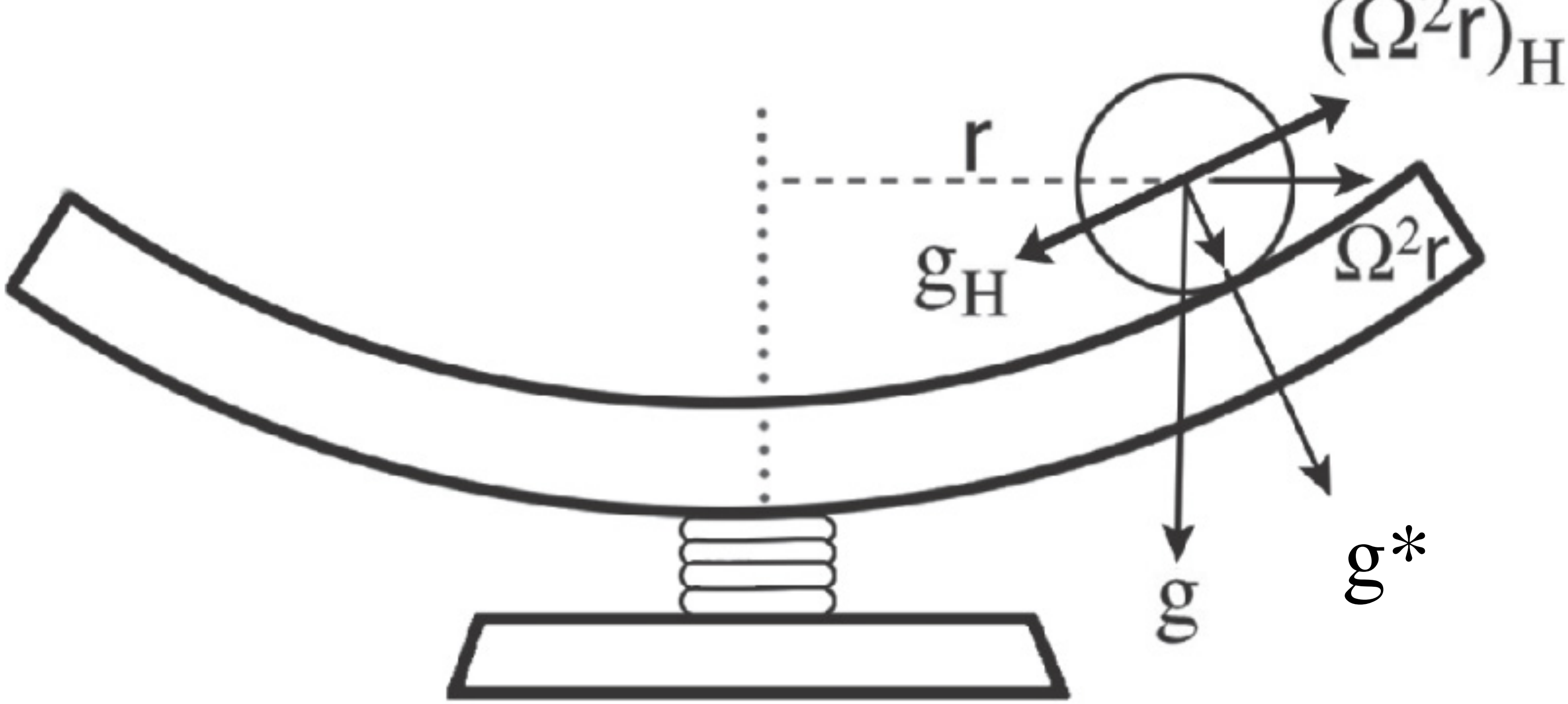
$$\frac{D\vec{u}}{Dt} = -\frac{1}{\rho}\vec{\nabla}p - 2\overrightarrow{\Omega} \times \vec{u} - \overrightarrow{g^{*}} + \overrightarrow{F_r}$$

Spherical Coordinates involves dealing with the components of Ω in CF

And leads to a re-definition of the vertical coordinate

In class exercise

incorporate centrifugal into g*



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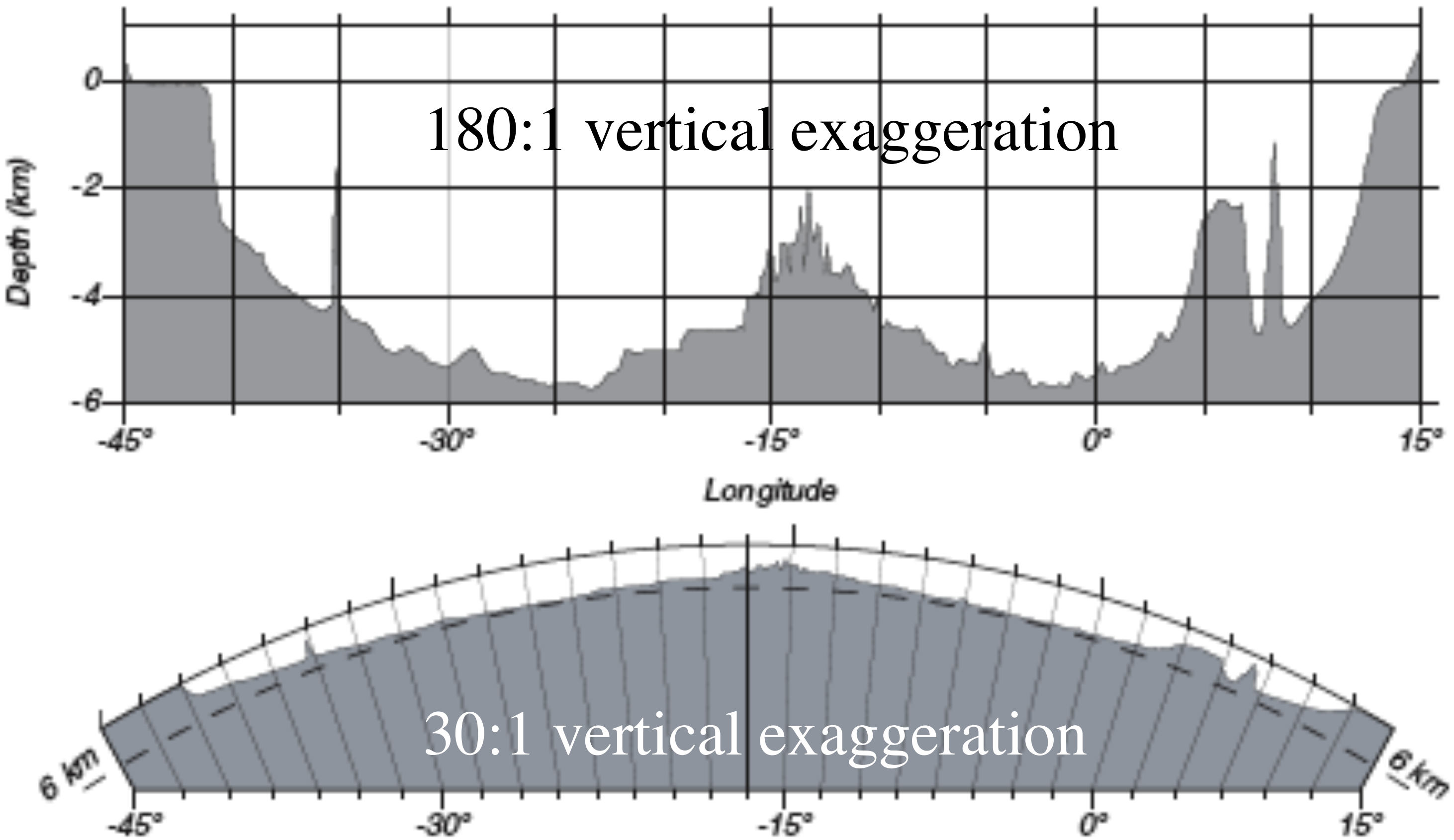
Navier Stokes puts these forces into a balance equation for conservation of momentum acting on a Lagrangian parcel in a rotating frame

$$\frac{D\vec{u}}{Dt} = -\frac{1}{\rho}\vec{\nabla}p - 2\overrightarrow{\Omega} \times \vec{u} - \overrightarrow{g^*} + \overrightarrow{F_r}$$

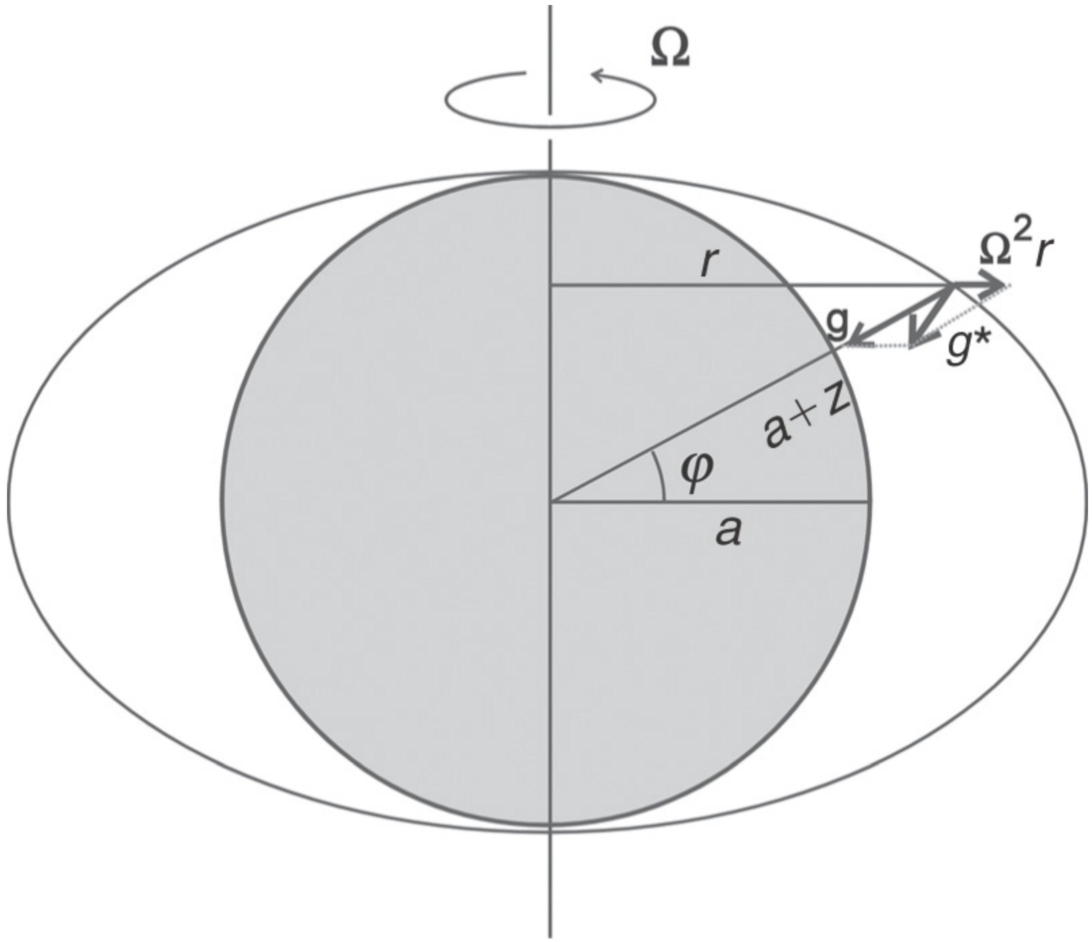
Width / Depth Ratio



Thus, a 2D approximation is often reasonable

Horizontal velocities (u,v) >> Vertical velocities (w)

Geopotential



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With z defined by geopotential

$$\frac{D\vec{u}}{Dt} = -\frac{1}{\rho}\vec{\nabla}p - 2\vec{\Omega}\times\vec{u} - \overrightarrow{\nabla\Phi} + \overrightarrow{F_r}$$

$$\frac{D\vec{u}}{Dt} = -\frac{1}{\rho}\vec{\nabla}p - 2\vec{\Omega}\times\vec{u} - \vec{g} + \overrightarrow{F_r}$$

Scaling

- What balance of forces dominates the motion?
- Base this determination on the SCALE of the terms
  - Length scale
  - Velocity scale
  - Time scale

In the open ocean (ignoring friction), what is the <u>dominant balance</u> in Navier-Stokes?

- $U \sim 10^{-1}$ m/s
- $W \sim 10^{-4}$ m/s
- $L \sim 10^{6}$ m
- $f \sim 10^{-4}$ 1/s
- $H \sim 10^{3}$ m
- $T \sim L/U$
- $P_o \sim 10^{7}$ Pa
- $\Delta p/\Delta L \sim 10^{-2}$ Pa/m

- $\rho' \sim 10^{0}$ kg/m^3
- $\rho_o \sim 10^{3}$ kg/m^3
- $\upsilon' \sim 10^{0}$-10^{2} (eddy diffusivity, horizontal)
- $\upsilon' \sim 10^{-3}$-10^{-5} m^2/s (eddy diffusivity, vertical)

$\upsilon_m \sim 10^{-6}$ m^2/s (molecular diffusivity)

Hydrostatic and Geostrophic Balances

Rossby Number

$$R_o = \frac{U}{fL}$$

- IF dominant balance in the Navier Stokes equation is the geostrophic balance, then Ro < 1
- In the ocean interior, Ro < 1 is usually a very good assumption

Continuity Equation (conservation of mass)

Incompressible -> conservation of volume

Continuity, compressible

$$\frac{1}{\rho}\frac{D\rho}{Dt}+\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

Continuity, incompressible (conservation of volume)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Boussinesq approximation

- Simplify the equations by recognizing the density variations are very small (~0.1%) in ocean
- However, for buoyancy calculations (i.e. where density is multiplied by g) its variations are important
  - Otherwise dp/dz would be constant, and horizontal pressure gradients couldn't develop – these are critical to dynamics

Conservation of Energy

$$\frac{Du}{Dt} - fv = -\frac{1}{\rho_o}\frac{\partial p}{\partial x} + F_x \qquad \text{with } f = 2\Omega\sin\varphi$$

$$\frac{Dv}{Dt} + fu = -\frac{1}{\rho_o}\frac{\partial p}{\partial y} + F_y$$

$$0 = -\frac{\partial p}{\partial z} - \rho g$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\frac{D\sigma}{Dt} = -\alpha_T\rho_o\frac{DT}{Dt} + \beta_S\rho_o\frac{DS}{Dt}$$

Combine equations with the right initial and boundary conditions

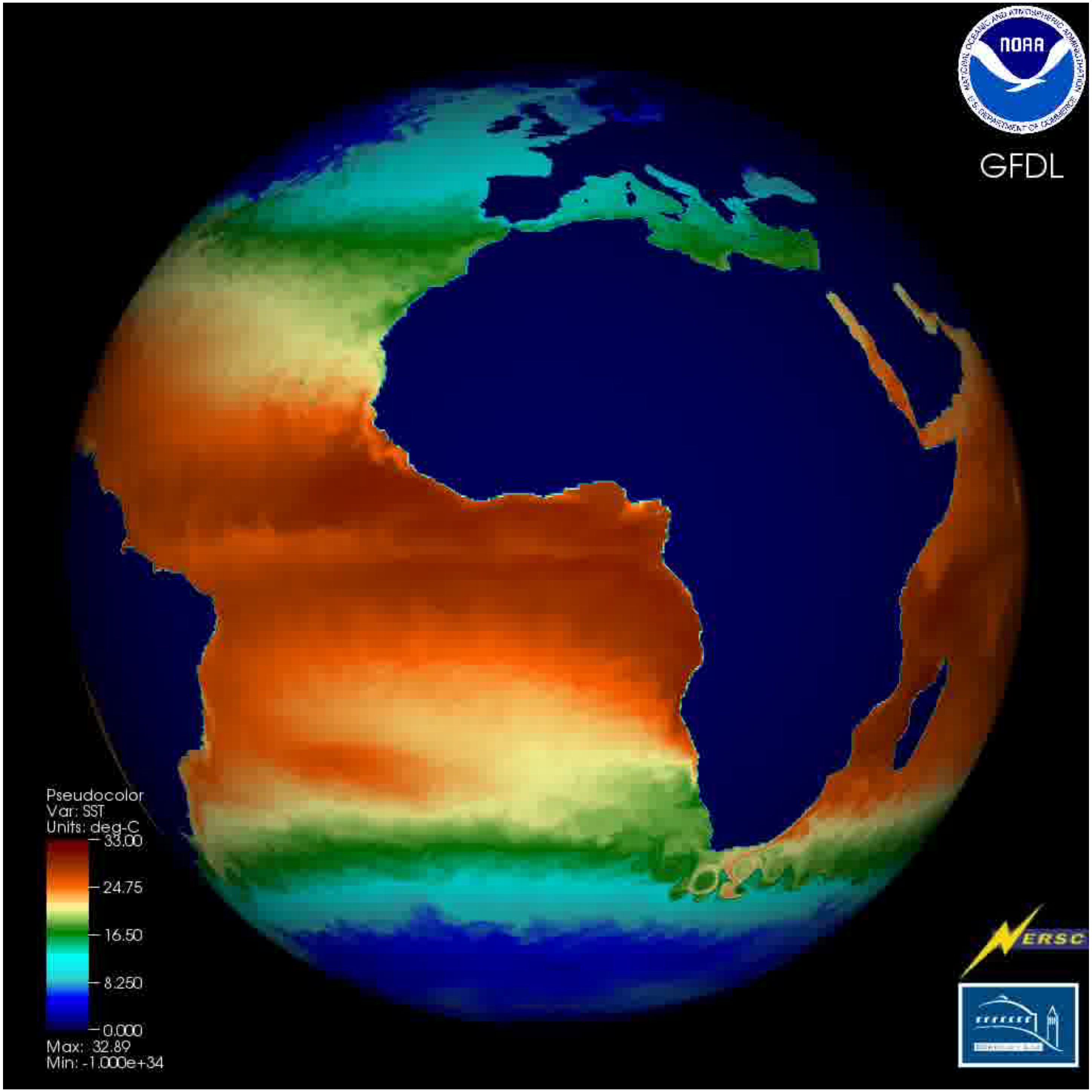
And you can capture the real ocean in a model!

1 year coupled climate simulation

Ocean: 0.1° to 0.25°

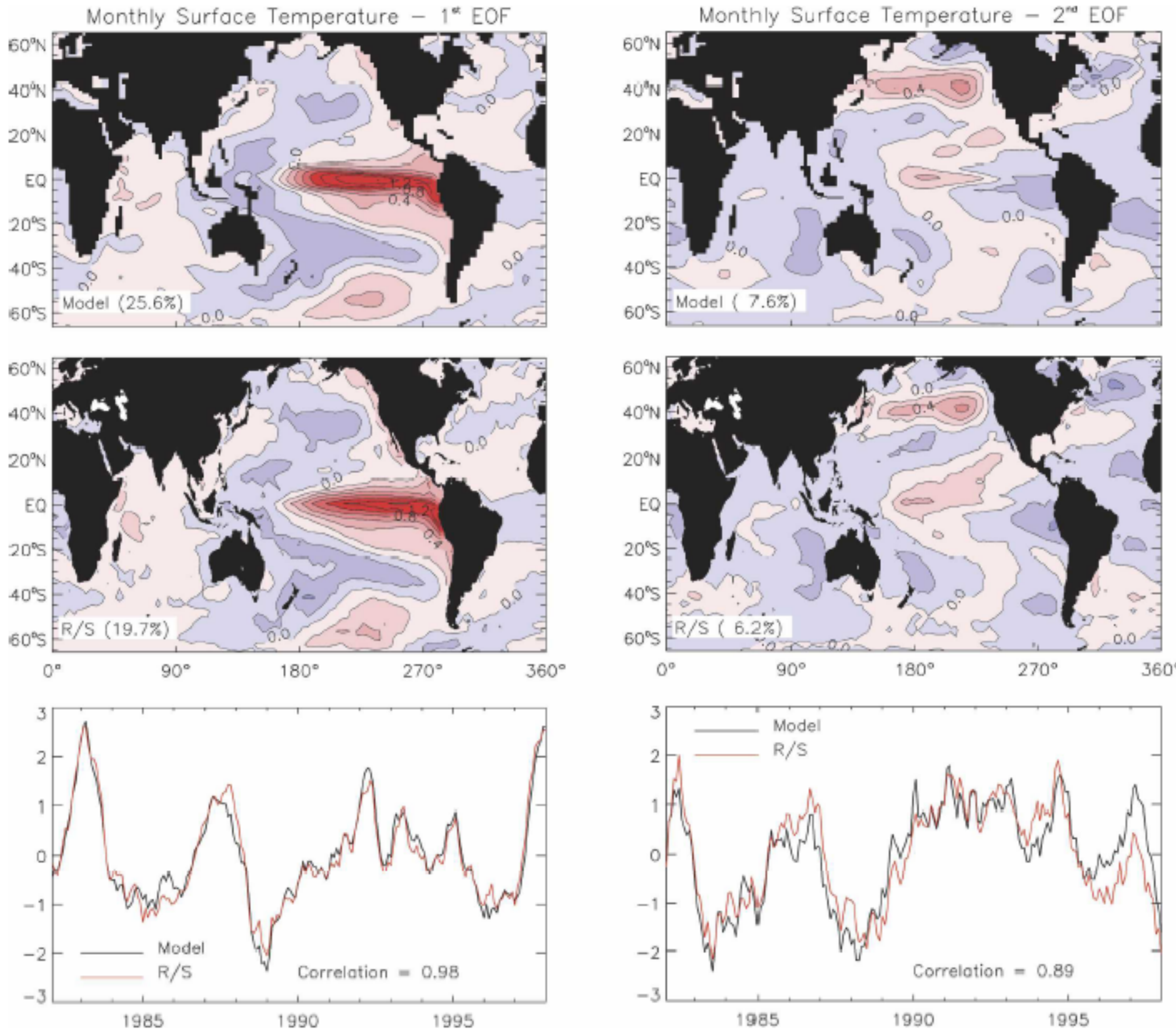
Atmosphere: 1°

NOAA GFDL



Model

Data

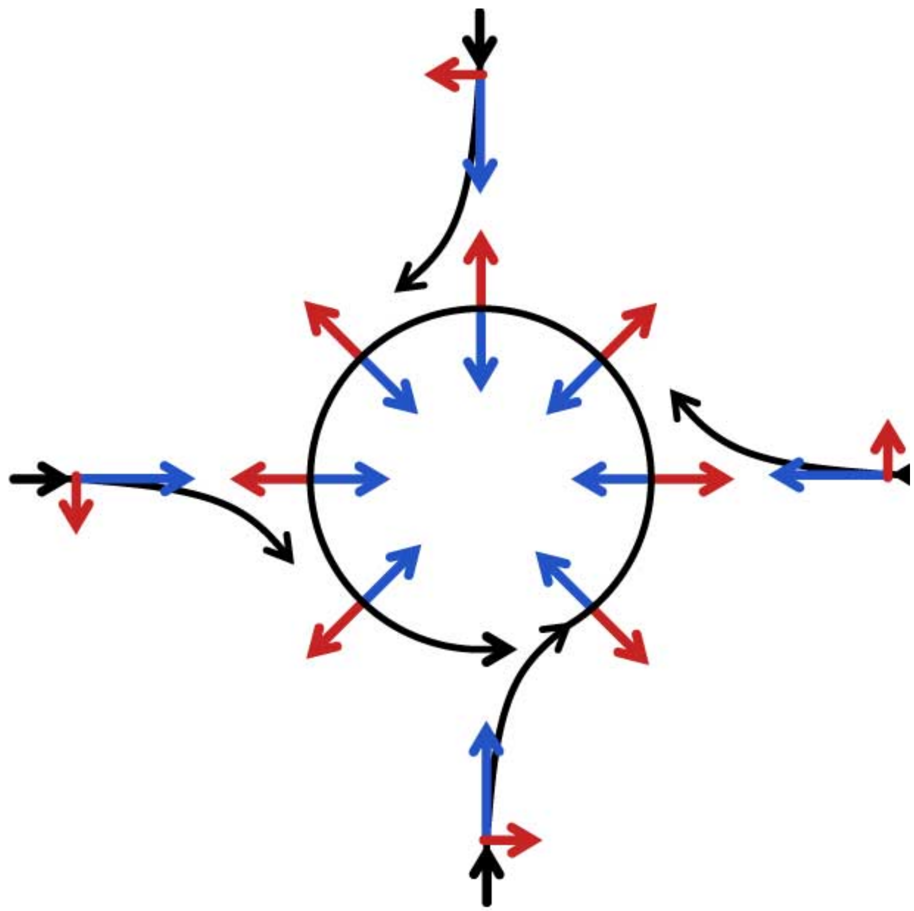


Doney et al. 2007

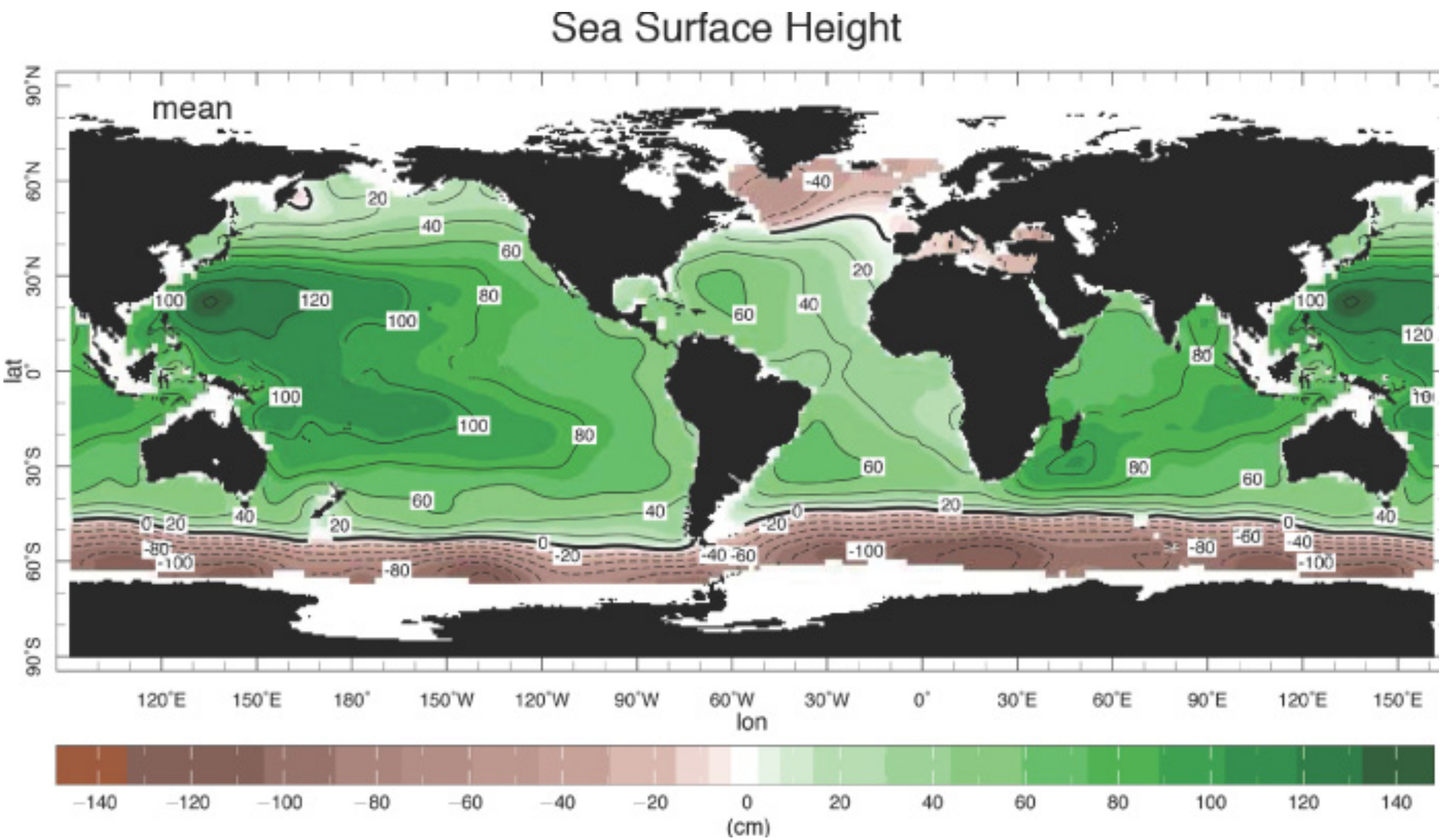
Balanced Flow

Geostrophic Balance:
Cyclonic system (a "low") in N. Hemisphere

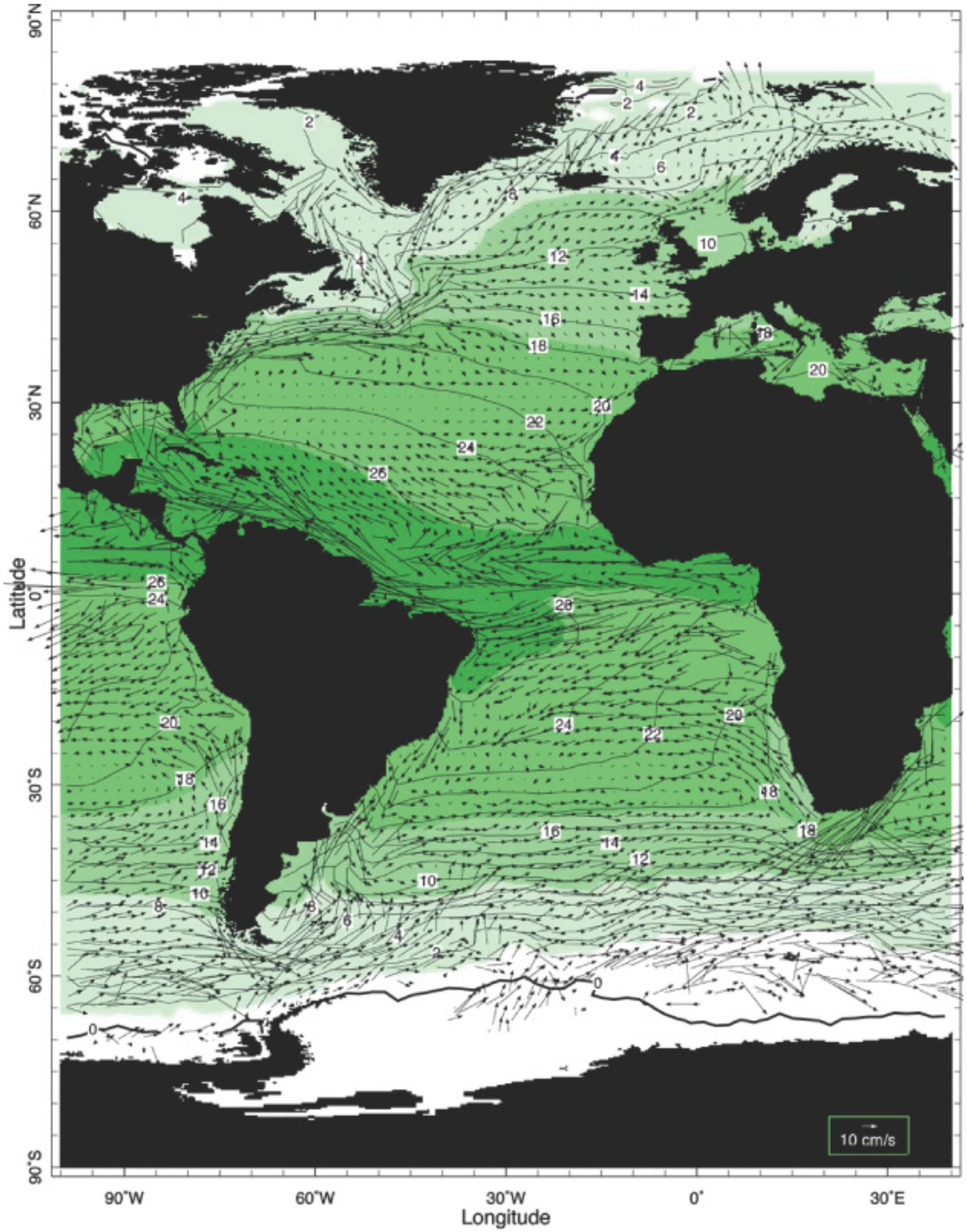
Inward flow due to PGF (blue force arrows) balance CF (red arrows) to create cyclonic flow (black arrows)



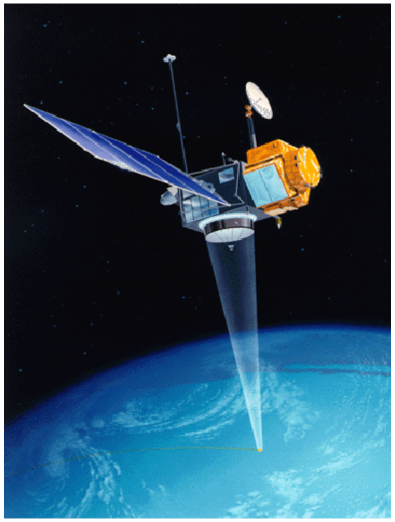
Applications to ocean currents



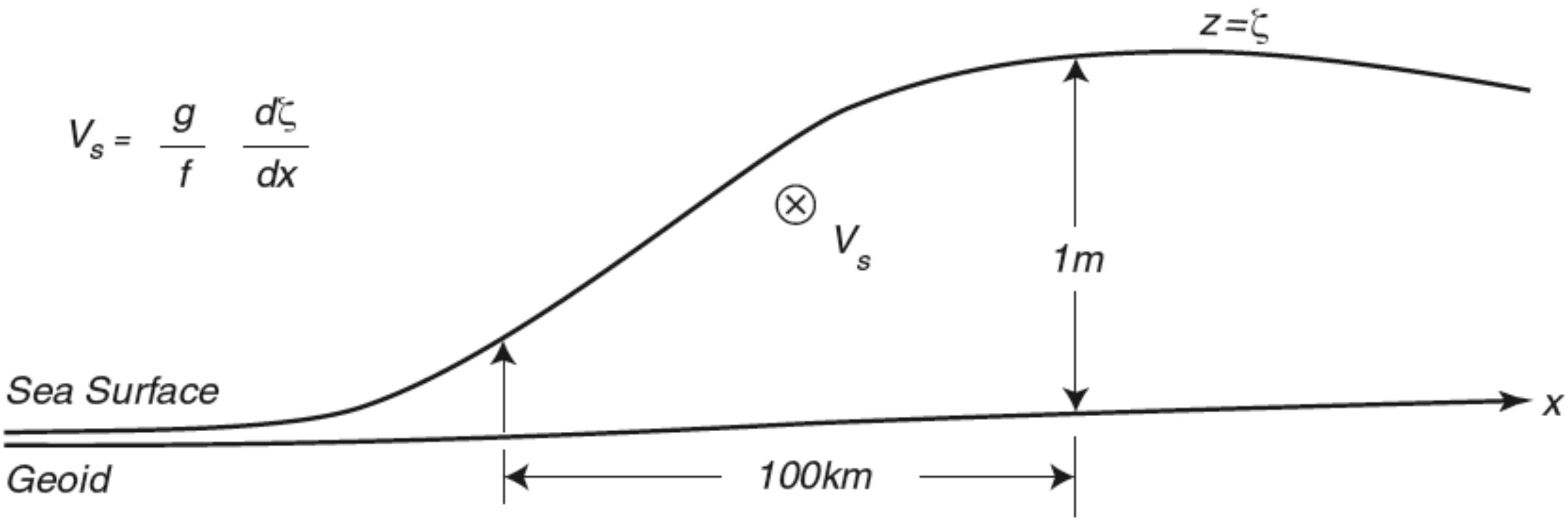
Sea Surface Height
mean
lat
90°N
60°N
30°N
0°
30°S
60°S
90°S
120°E
150°E
180°
150°W
120°W
90°W
60°W
30°W
0°
30°E
60°E
90°E
120°E
150°E
lon
−140
−120
−100
−80
−60
−40
−20
0
20
40
60
80
100
120
140
(cm)



(Data courtesy of Maximenko and Niiler (personal communication, 2003).)



Dynamic Topography to get surface currents



Interior geostrophic flow

- At every level, geostrophy must apply
  - Coriolis and horizontal pressure gradient in balance

- Find flow by integrating density structure to find pressure – this is the relative velocity
- Add relative and surface velocity

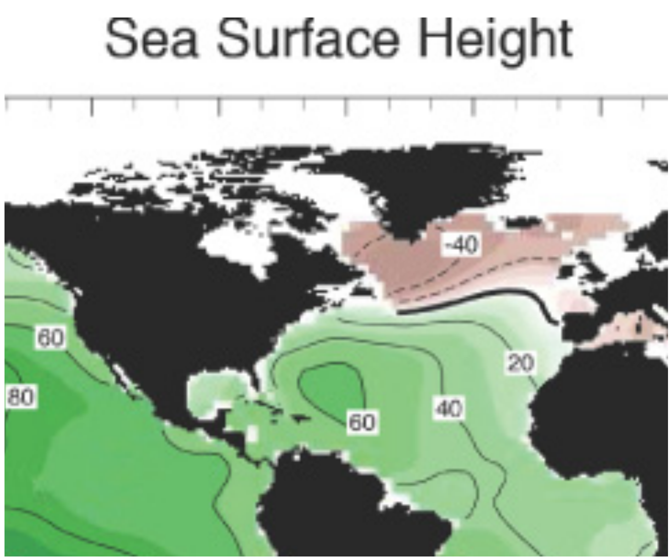
Interior Geostrophic flow

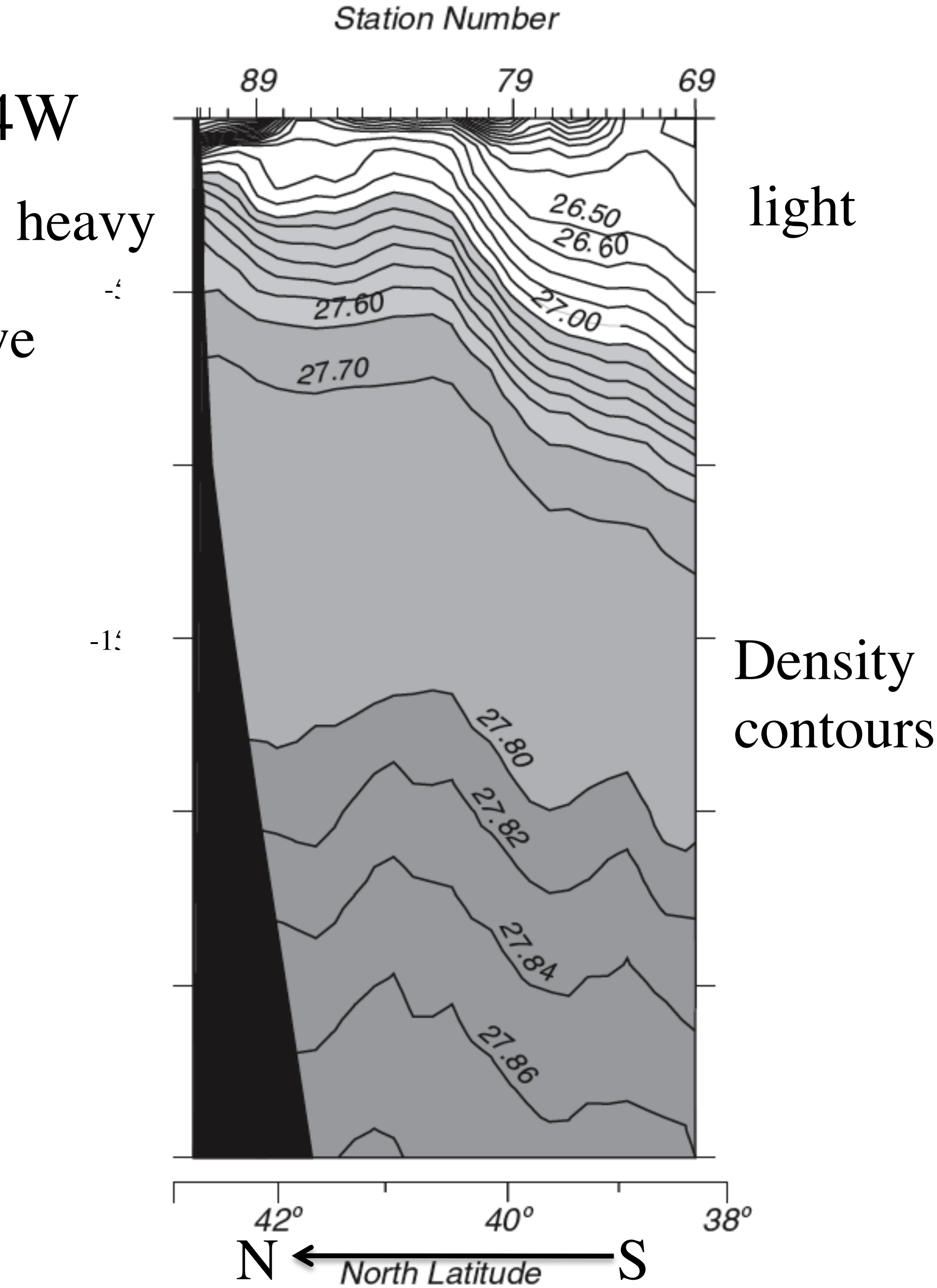
$$\vec{u} = \frac{g}{f\rho_o}\left[\langle\rho\rangle\hat{k}\times\nabla\eta + (\eta - z)\hat{k}\times\nabla\langle\rho\rangle\right]$$

$$\vec{u} \approx \frac{g}{f}\hat{k}\times\nabla\eta + \frac{g(\eta - z)}{f\rho_o}\hat{k}\times\nabla\langle\rho\rangle$$

Gulf Stream Section at 64W

Which direction of flow do we expect?





Make a prediction for the Tank Experiment

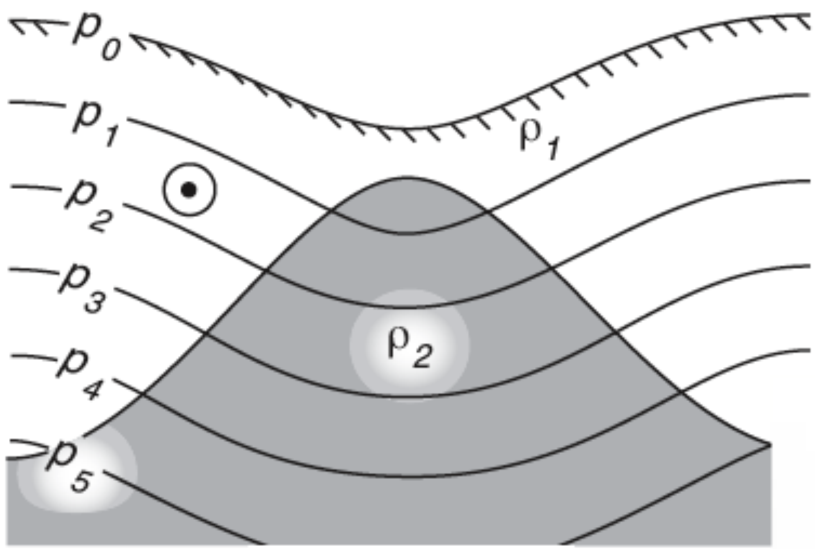
- Sketch the mound of fluid that will occur after can is pulled out.
- Apply geostrophy.
- What direction is the of flow, surface and bottom?

What is observed?

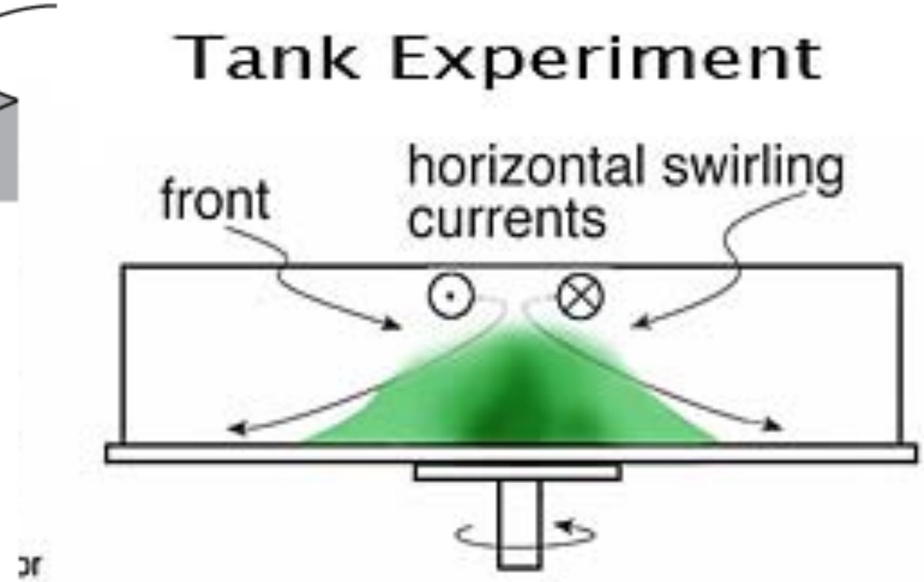
Why is this different from your prediction?

What else is going on that you might not have accounted for?

Consider the pressure distribution



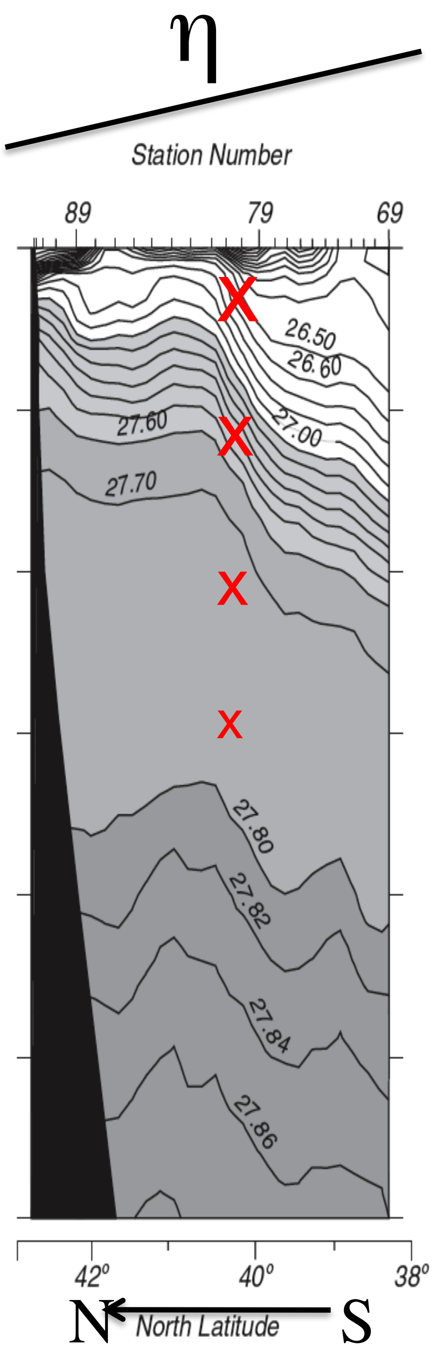
Cold-core ring



The surface height deviation is key!

- Due to the subsurface density distribution, the relative velocity is <u>opposite</u> the surface velocity
- Sum of the relative and surface velocity that gives the velocity at a depth: Decays down

$$\vec{u} \approx \frac{g}{f}\hat{k} \times \nabla \eta + \frac{g(\eta - z)}{f\rho_o}\hat{k} \times \nabla \langle \rho \rangle$$



heavy

light $u_g = -\dfrac{g}{f}\dfrac{\partial \eta}{\partial y} - \dfrac{g(\eta - z)}{f\rho_0}\dfrac{\partial \langle \rho \rangle}{\partial y}$

+

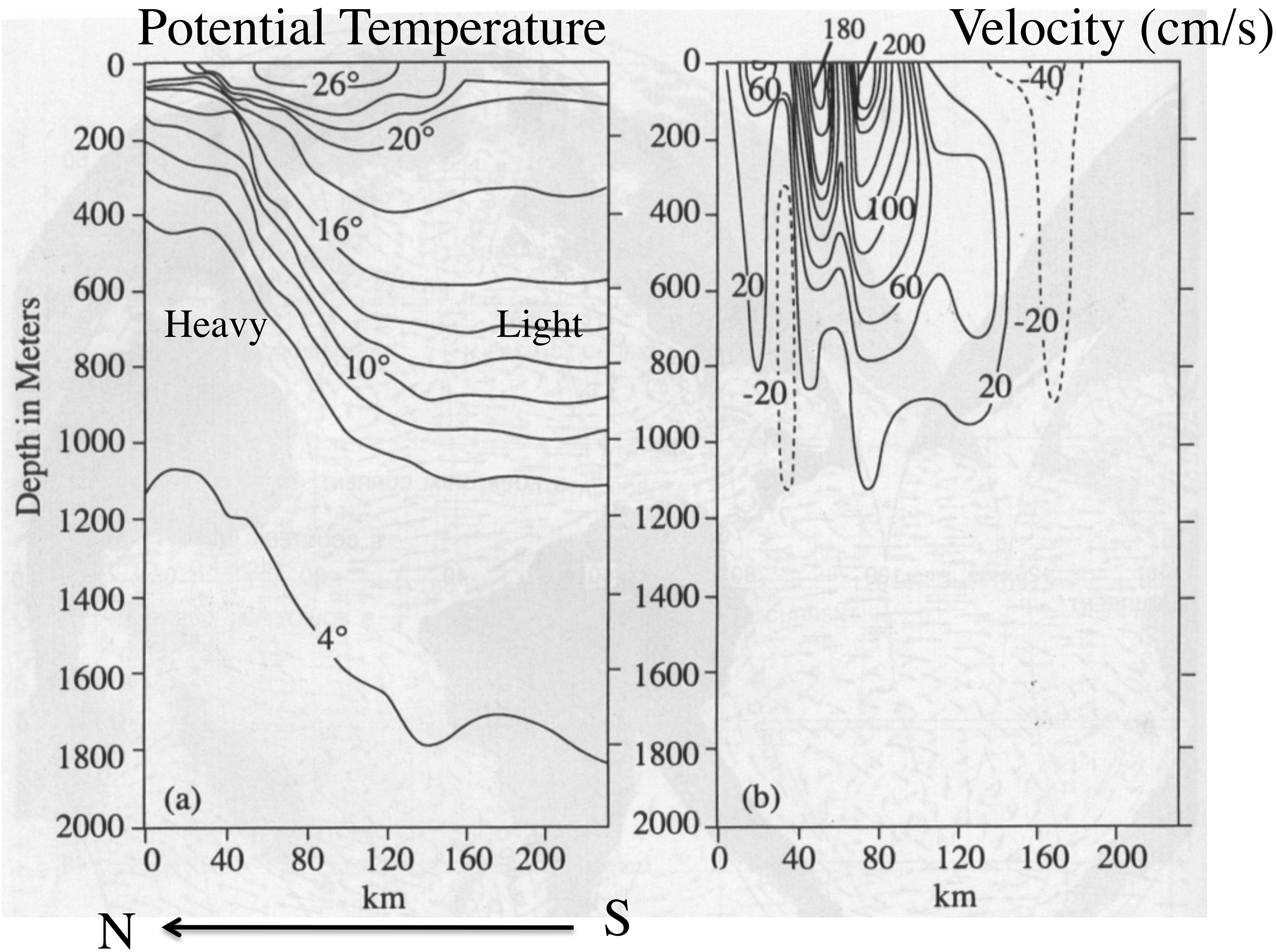
−

$\Delta y > 0$

$\Delta \eta < 0$

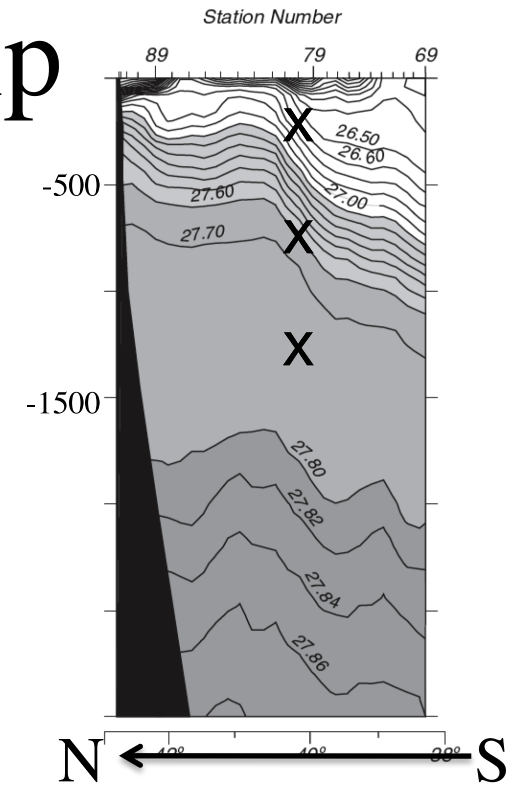
$\Delta \langle \rho \rangle > 0$

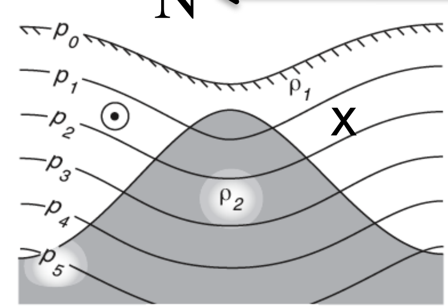
Gulf Stream section: 38N, 68W



A counter-intuitive density / pressure relationship

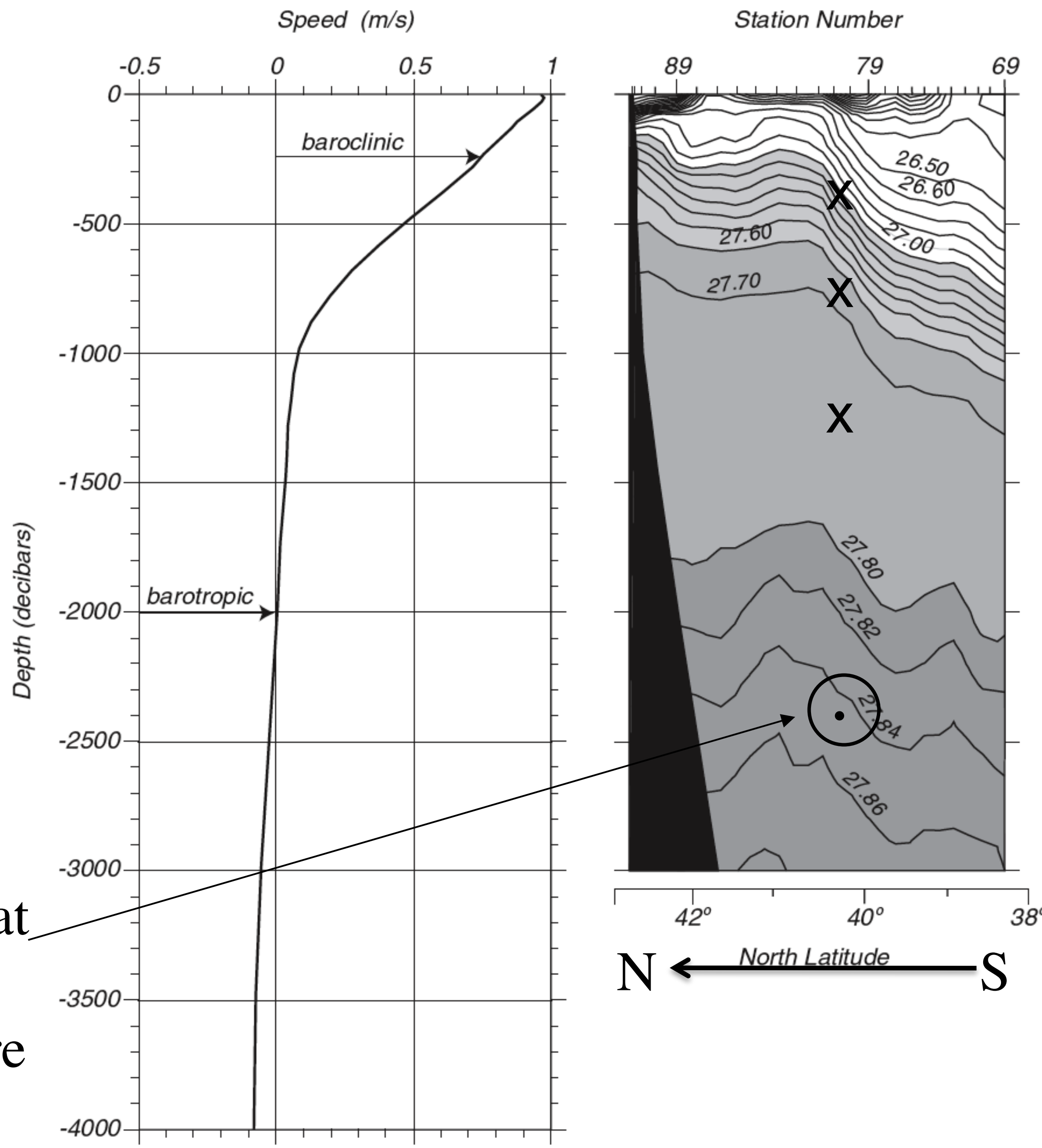
- At depth, the density and pressure distribution are not aligned
- WHY?
  - Due to the surface deformation
  - Until the level of no motion is reached, the surface deformation exerts a pressure anomaly that overcomes the internal density distribution





Cold-core ring

Gulf Stream Section at 64W



Direction changes at depth because here density and pressure are aligned

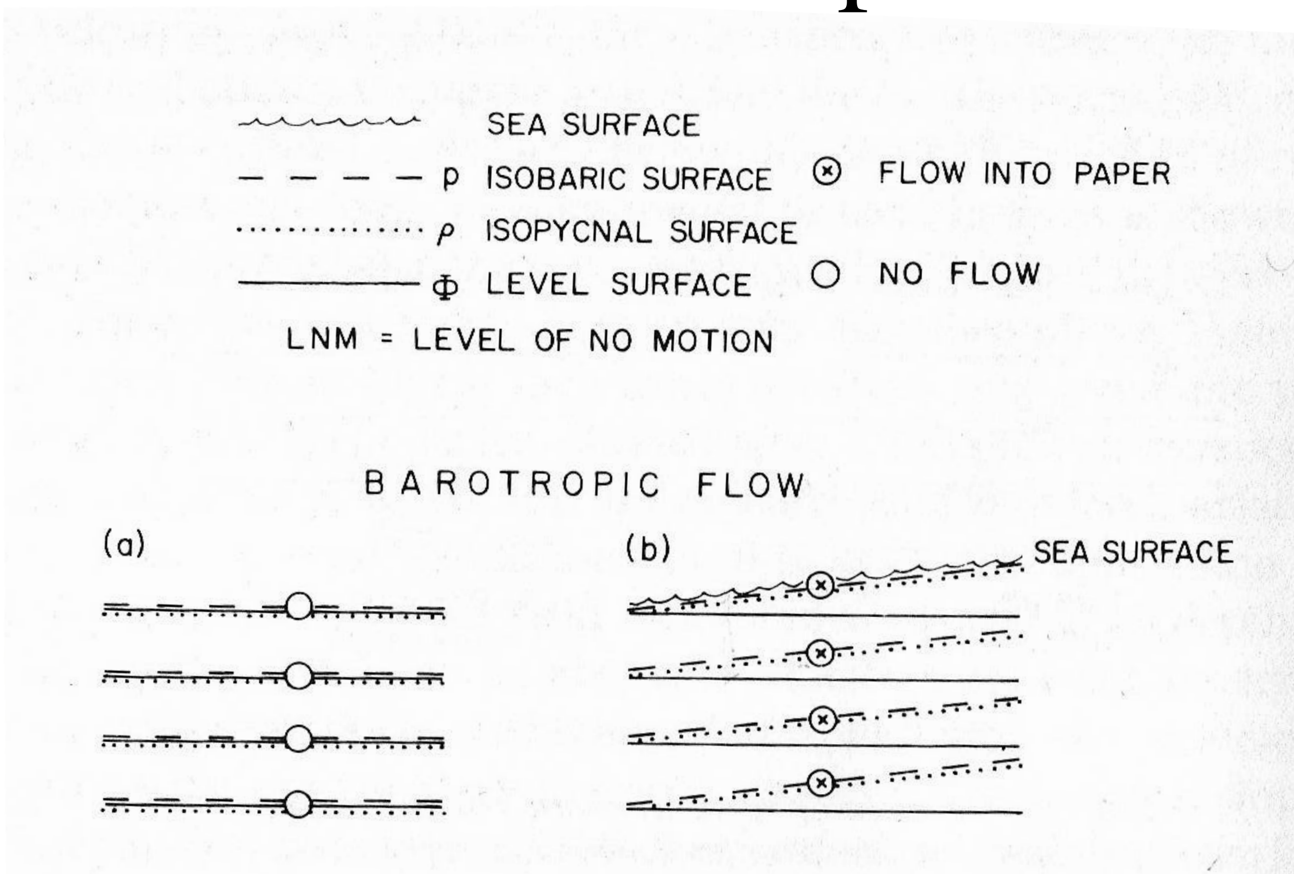
Baroclinic vs. Barotropic

Barotropic

$$\vec{u} \approx \frac{g}{f}\hat{k} \times \nabla \eta + \frac{g(\eta - z)}{f\rho_o}\hat{k} \times \nabla \langle \rho \rangle$$

1. Density, pressure parallel to geoid = no flow
2. If homogeneous (ρ = constant)
   1. Second term on RHS = 0
   2. Surface flow extends to depth (OR the influence of depth can extend to the surface! Taylor Columns.)
3. If isopycnals and isobars are parallel
   1. i.e. no vertical change in horizontal pressure <u>gradient</u>

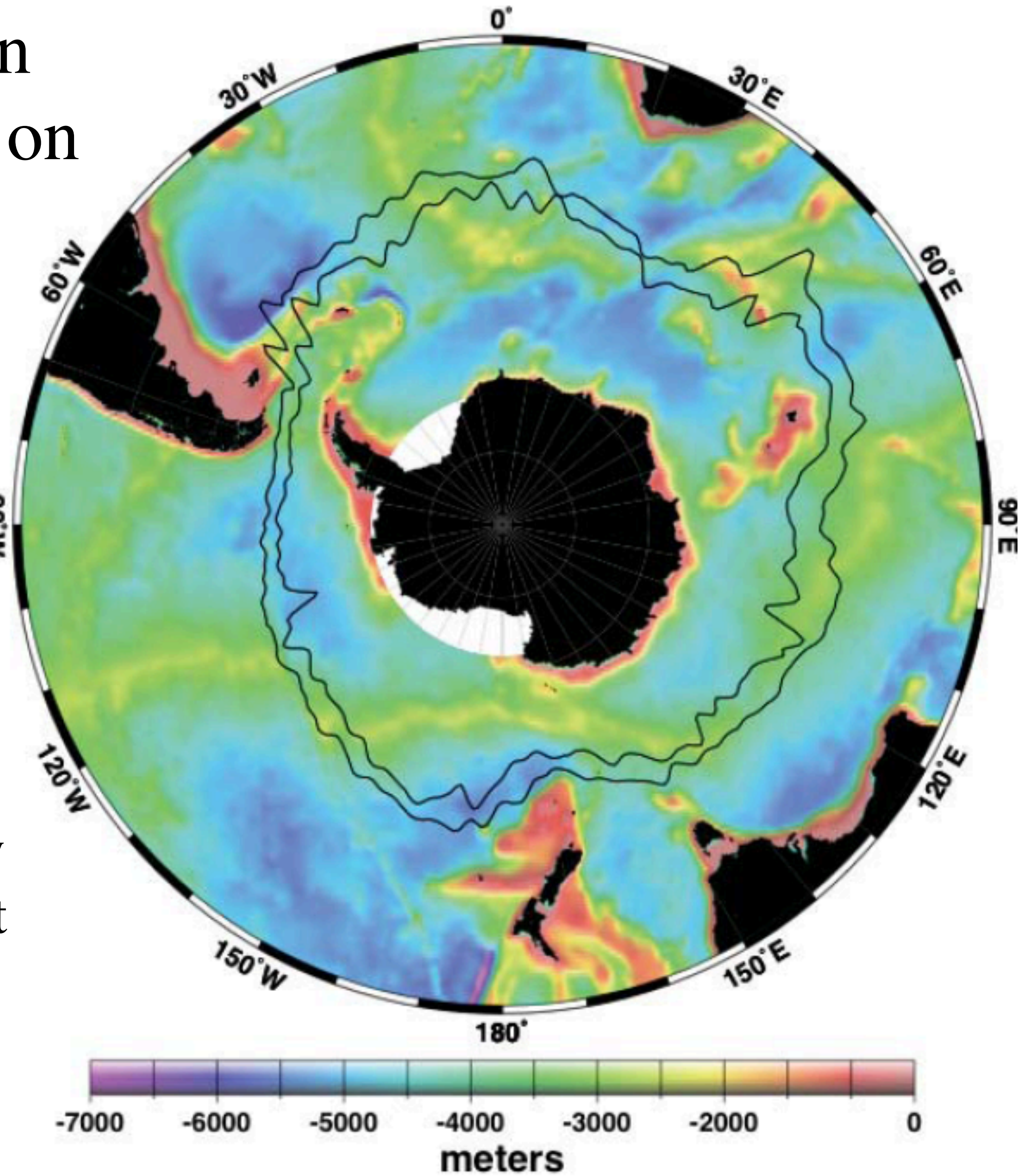
Barotropic



SEA SURFACE
p ISOBARIC SURFACE
FLOW INTO PAPER
ρ ISOPYCNAL SURFACE
NO FLOW
Φ LEVEL SURFACE
LNM = LEVEL OF NO MOTION
BAROTROPIC FLOW
(a)
(b)
SEA SURFACE

ACC path (between lines) superimposed on bathymetry

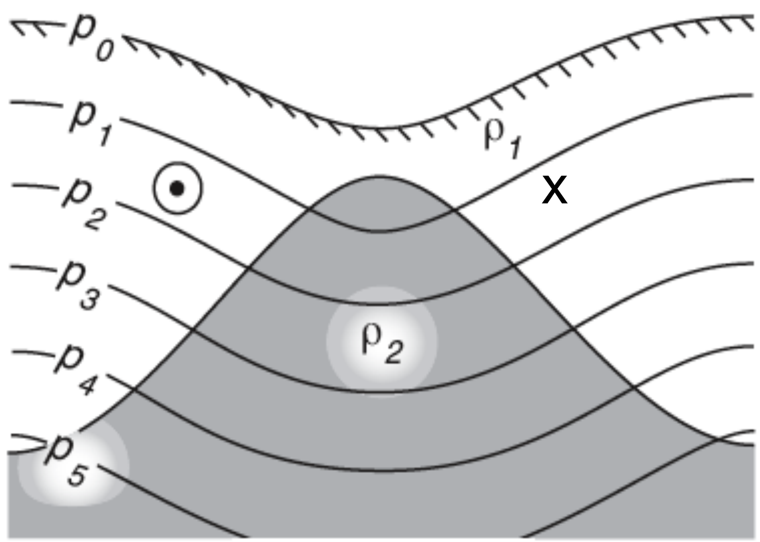
(Gille et al. 2004)



At first order, ACC is barotropic and is steered by topography. Remember that high latitude regions are weakly stratified

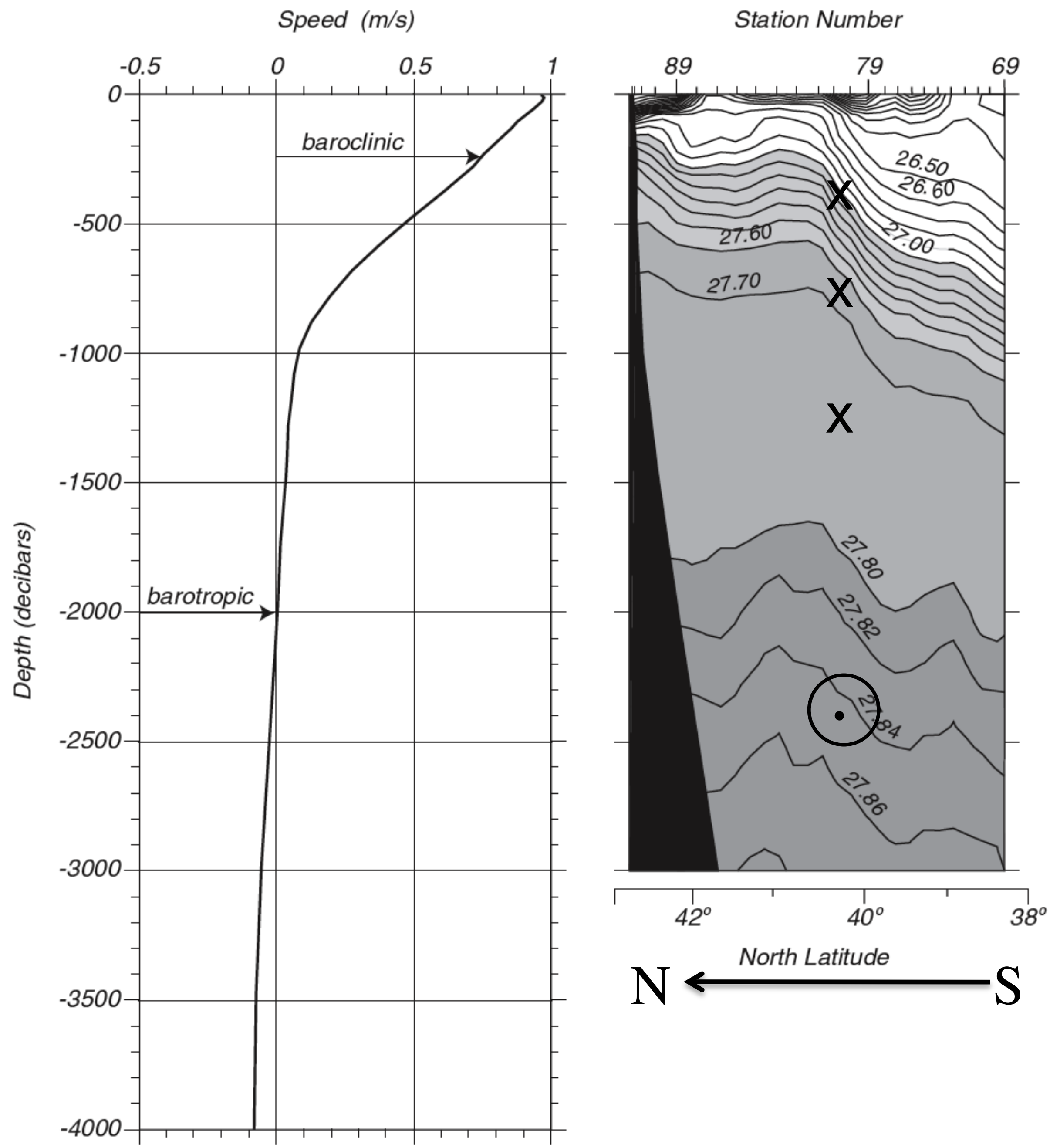
Baroclinic

- When isopycnals and isobars are not parallel to each other



Cold-core ring

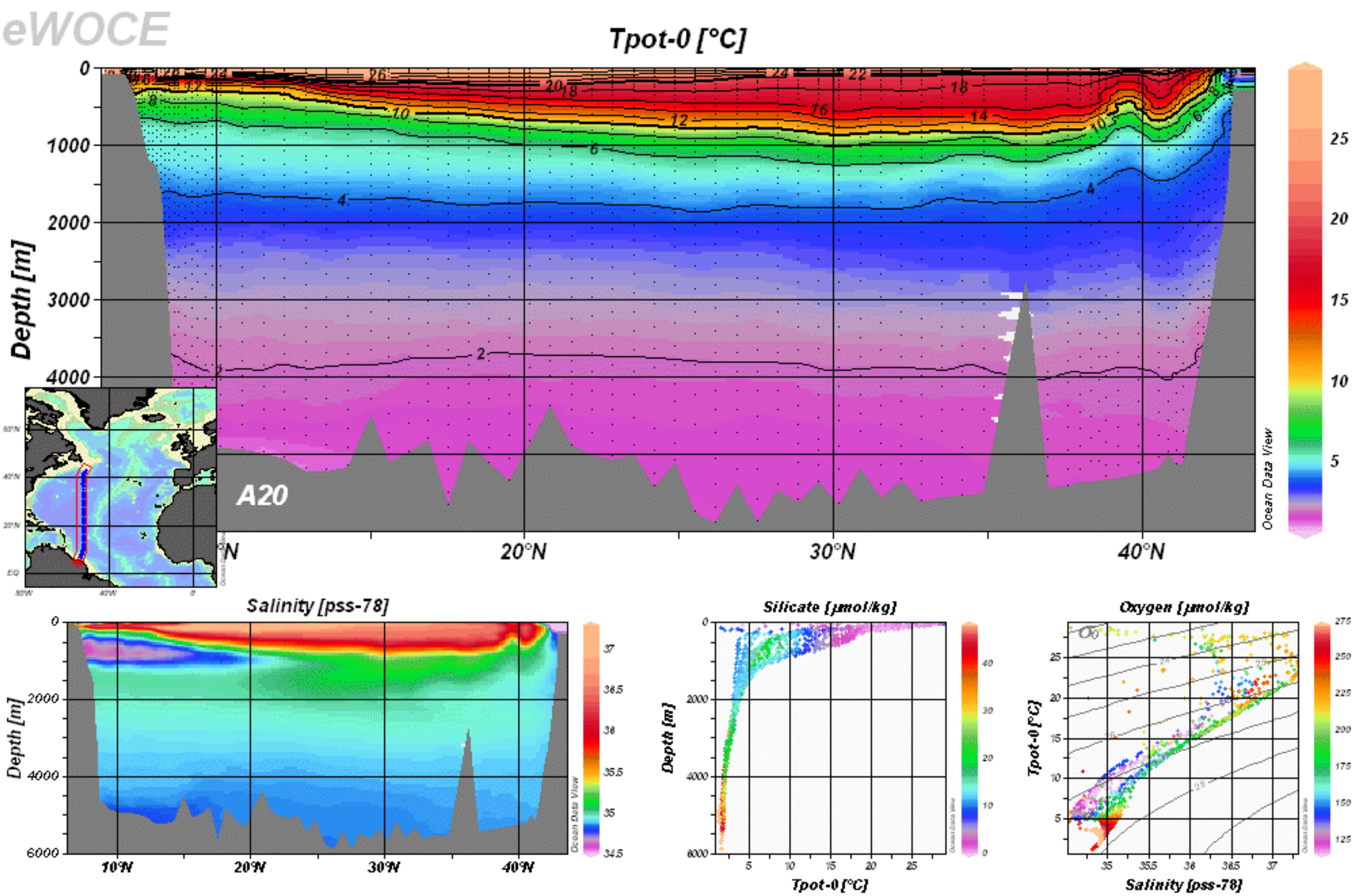
Gulf Stream Section at 64W



For a baroclinic fluid, thermal wind allows estimation of the vertical shear in the flow

$$\frac{\partial u_g}{\partial z} = \frac{g}{f\rho_o}\frac{\partial \rho}{\partial y} = -\frac{g\alpha_T}{f}\frac{\partial T}{\partial y}$$

$$\frac{\partial v_g}{\partial z} = -\frac{g}{f\rho_o}\frac{\partial \rho}{\partial x} = \frac{g\alpha_T}{f}\frac{\partial T}{\partial x}$$


eWOCE
Tpot-0 [°C]
Depth [m]
A20
20°N
30°N
40°N
Ocean Data View
Salinity [pss-78]
Silicate [µmol/kg]
Tpot-0 [°C]
Oxygen [µmol/kg]
Salinity [pss-78]

Gulf Stream section: 38N, 68W