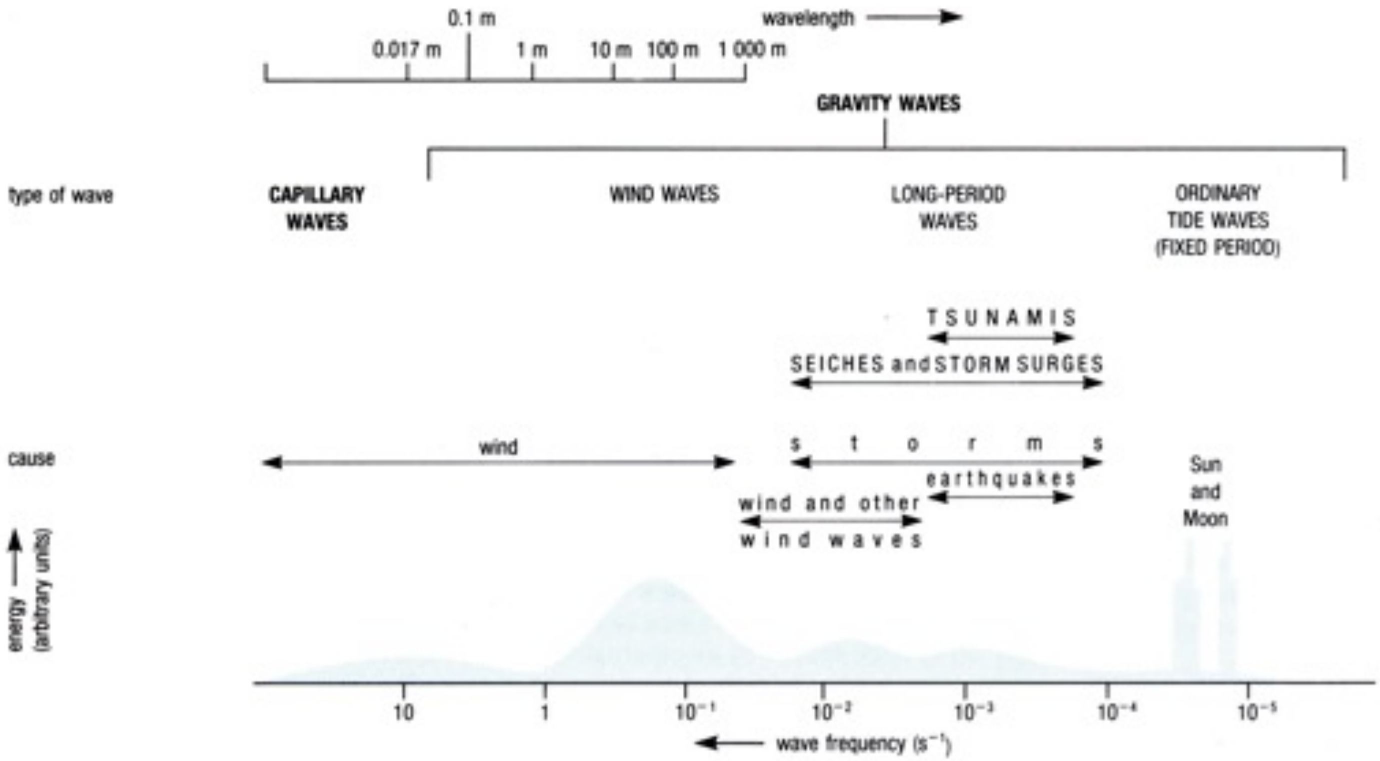
Waves

AOS660, Professor McKinley

14 November 2013

- Wave basics, a review
  - Dispersion relationship
  - Dispersive vs. Non-dispersive
- Examples
  - Shallow and Deep water Waves
    - Surface
    - Internal
    - Tsunami
  - Planetary Waves
    - Kelvin
    - Rossby



wavelength
0.1 m
0.017 m
1 m
10 m
100 m
1 000 m
GRAVITY WAVES
type of wave
CAPILLARY WAVES
WIND WAVES
LONG-PERIOD WAVES
ORDINARY TIDE WAVES (FIXED PERIOD)
TSUNAMIS
SEICHES and STORM SURGES
cause
wind
storms
earthquakes
wind and other wind waves
Sun and Moon
energy (arbitrary units)
10
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
wave frequency (s^{-1})

What is a wave?

- Waves transfer energy without transfer of mass
- To exist, waves require
  - Inertia – set up by an initial disturbance
  - Restoring force

Particles have orbital trajectory

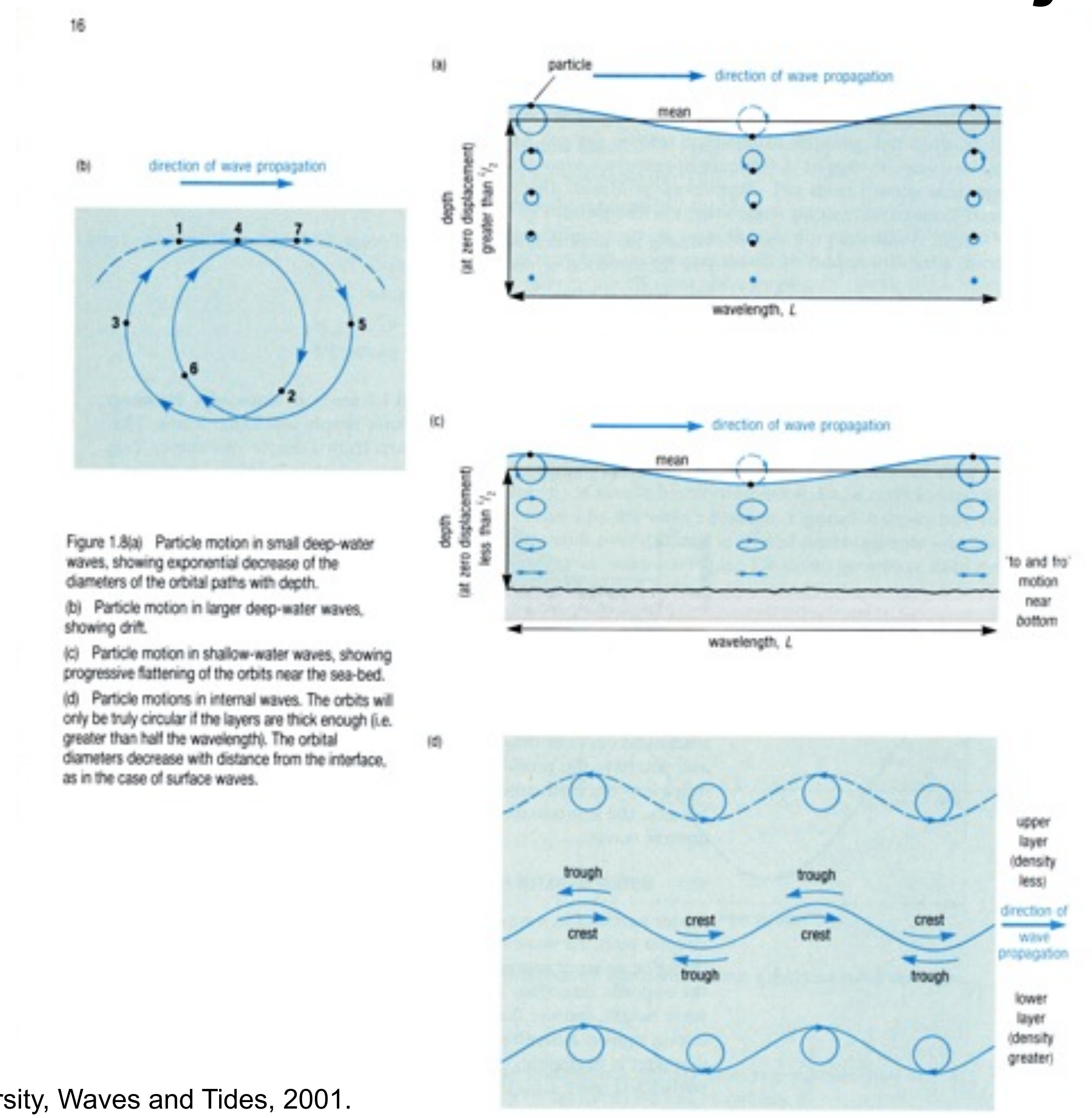


Figure 1.8(a) Particle motion in small deep-water waves, showing exponential decrease of the diameters of the orbital paths with depth.

(b) Particle motion in larger deep-water waves, showing drift.

(c) Particle motion in shallow-water waves, showing progressive flattening of the orbits near the sea-bed.

(d) Particle motions in internal waves. The orbits will only be truly circular if the layers are thick enough (i.e. greater than half the wavelength). The orbital diameters decrease with distance from the interface, as in the case of surface waves.

The Open University, Waves and Tides, 2001.
Figure 1.8

Wave dimensions

- Space
  - Wavelength (L)
  - Wavenumber (e.g. $k = 2\pi/L$)
  - Amplitude (A)
- Time
  - Period (T) (s)
  - Wave frequency f (1/s) or Angular Frequency σ (rad/s)
- Velocity
  - Phase speed (c), Group velocity (c_g) (m/s)

Dispersion relation

- Relates frequency to wave number
  - Though often given as speed to wave number relationship
- Fundamental property of the wave form!
  - Tells how the restoring force acts

Deriving the shallow water dispersion relationship

- Write down the equation of motion, shallow water
  - Ignore rotation, friction
  - Pressure gradient in terns of surface height
- Apply the perturbation method to consider small variations around the mean state
- Develop a second order DE for the surface height (use continuity)
- Assume surface height has a wave-like solution (e.g. $h' = Ae^{ik(x-ct)}$)
- Plug into the 2nd order DE and solve

Shallow water gravity wave speed, surface, no mean flow

ρ_1=1000 kg/m^3

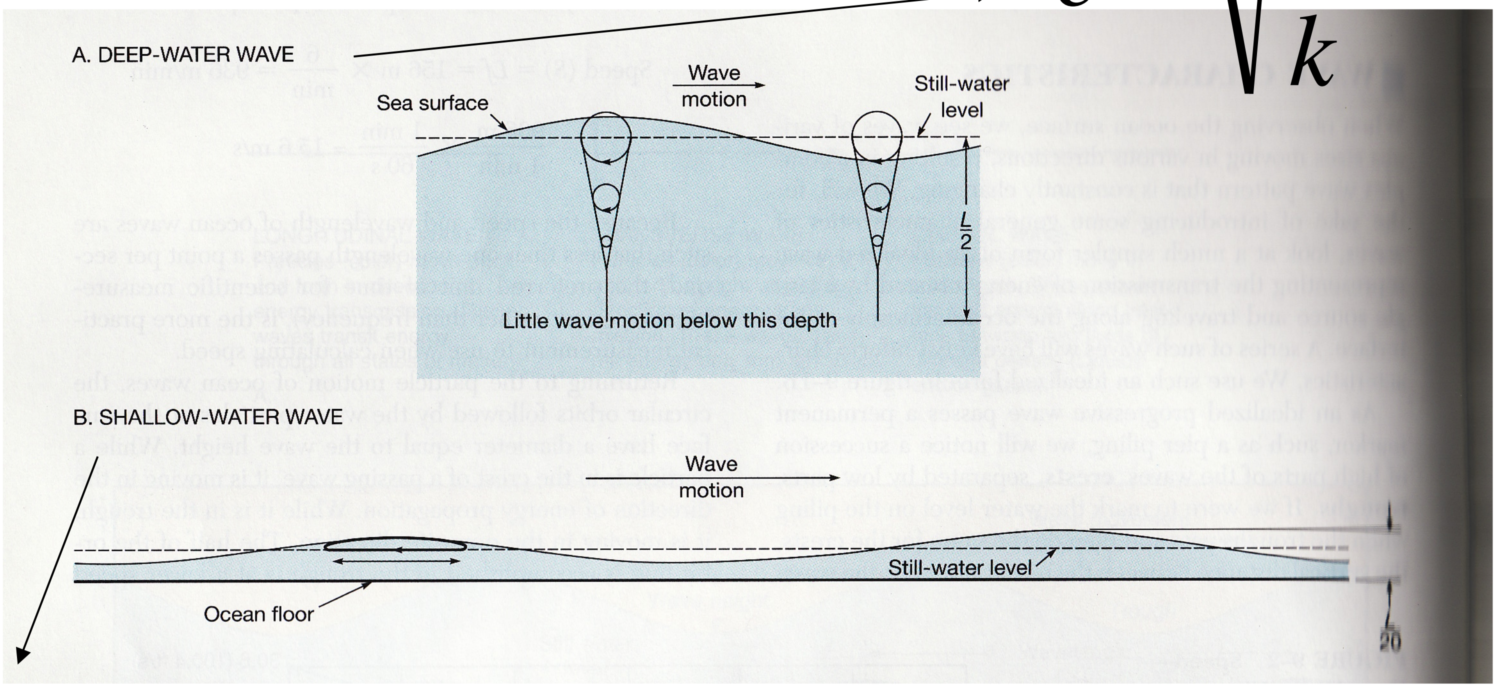
ρ_2=1 kg/m^3

$\rho' = \rho_1 - \rho_2 = 999$ kg/m^3

$\rho'/\rho_1 \sim 1$

$$c = \sqrt{gH}$$

$$c = \sqrt{\frac{g}{k}}$$



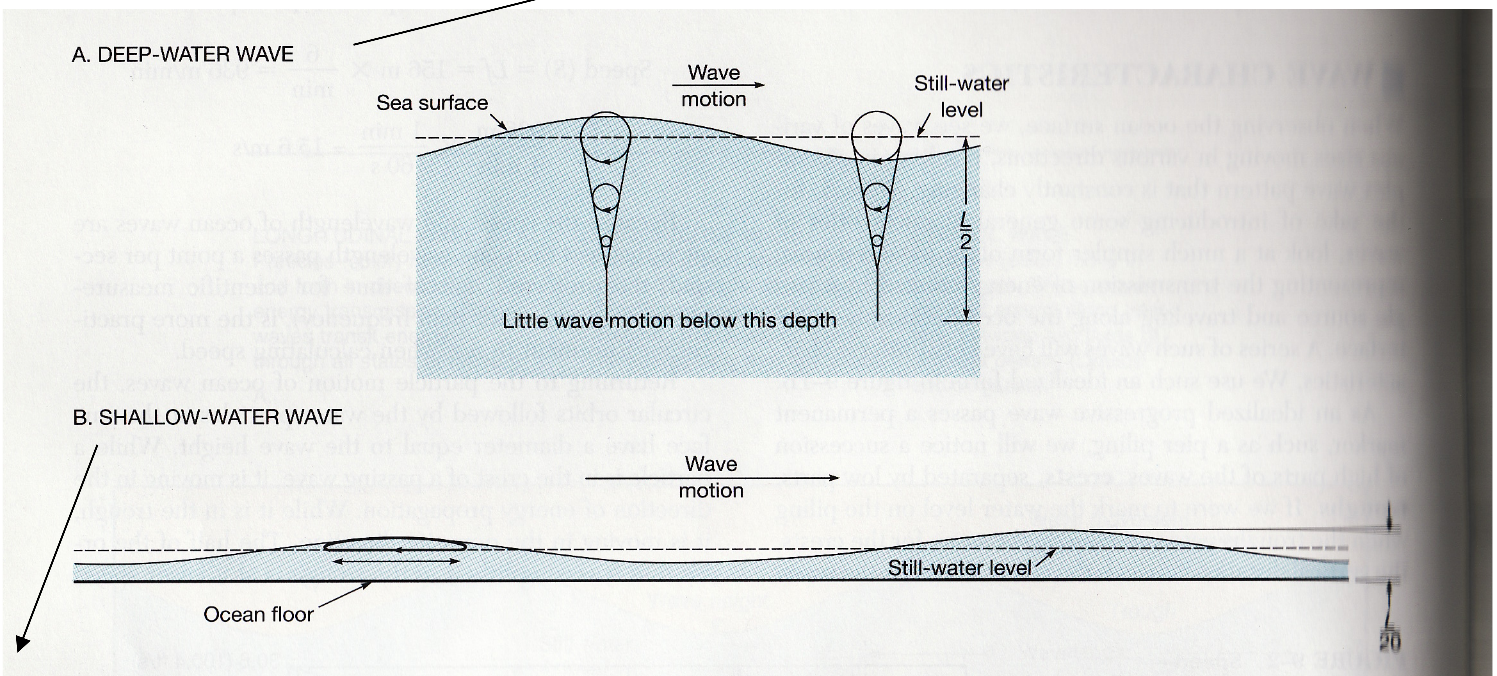
$$c = \sqrt{gH}$$

Dispersive vs. Non-dispersive

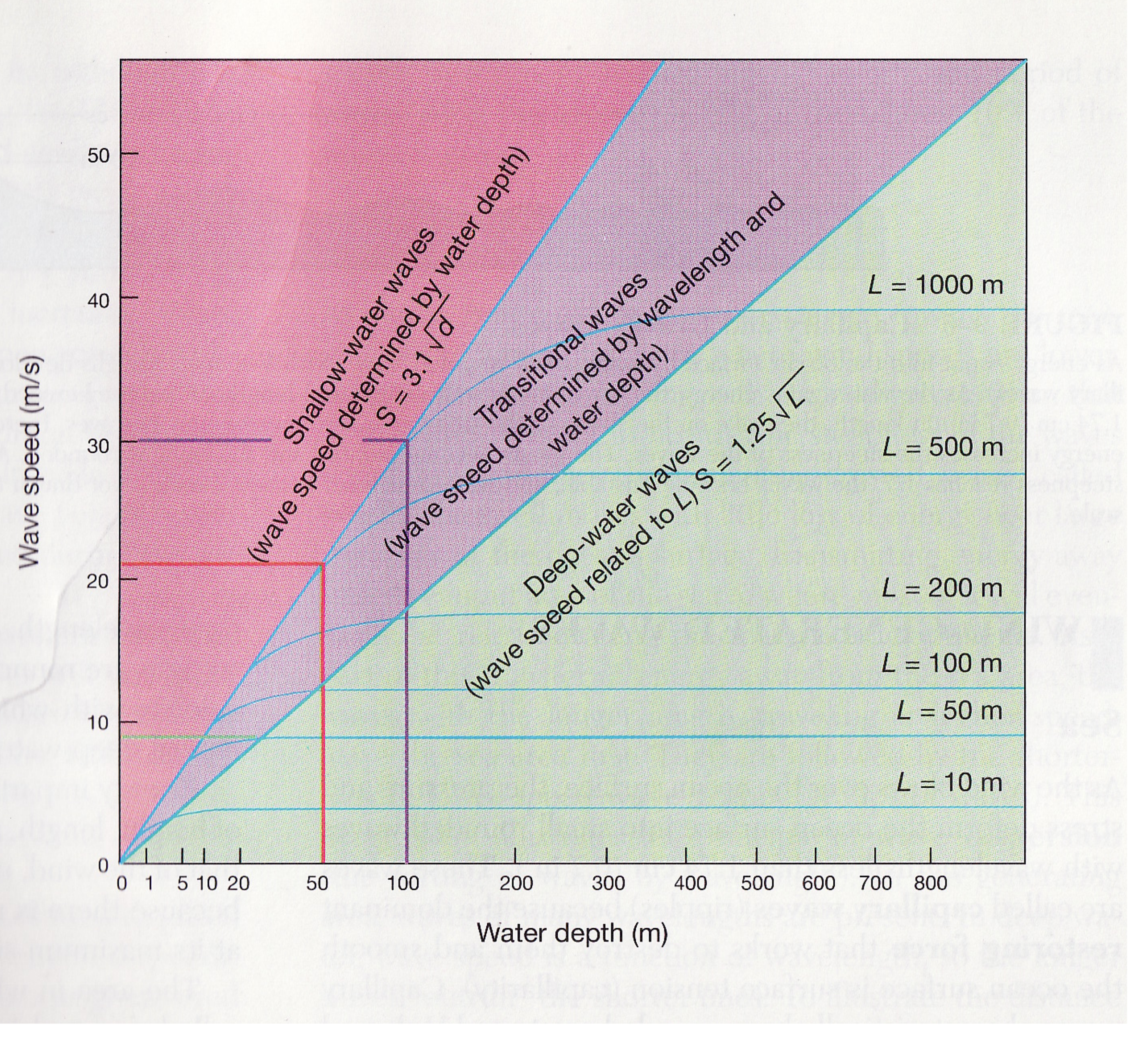
- If the wave speed depends on the wave frequency (or wave length), then the wave speed changes depending on the wave form
- If no such dependence, all waves of this type (no matter their size), propagate at the same speed

Which of these waves is dispersive?

$$c = \sqrt{\frac{g}{k}} = \sqrt{\frac{gL}{2\pi}}$$



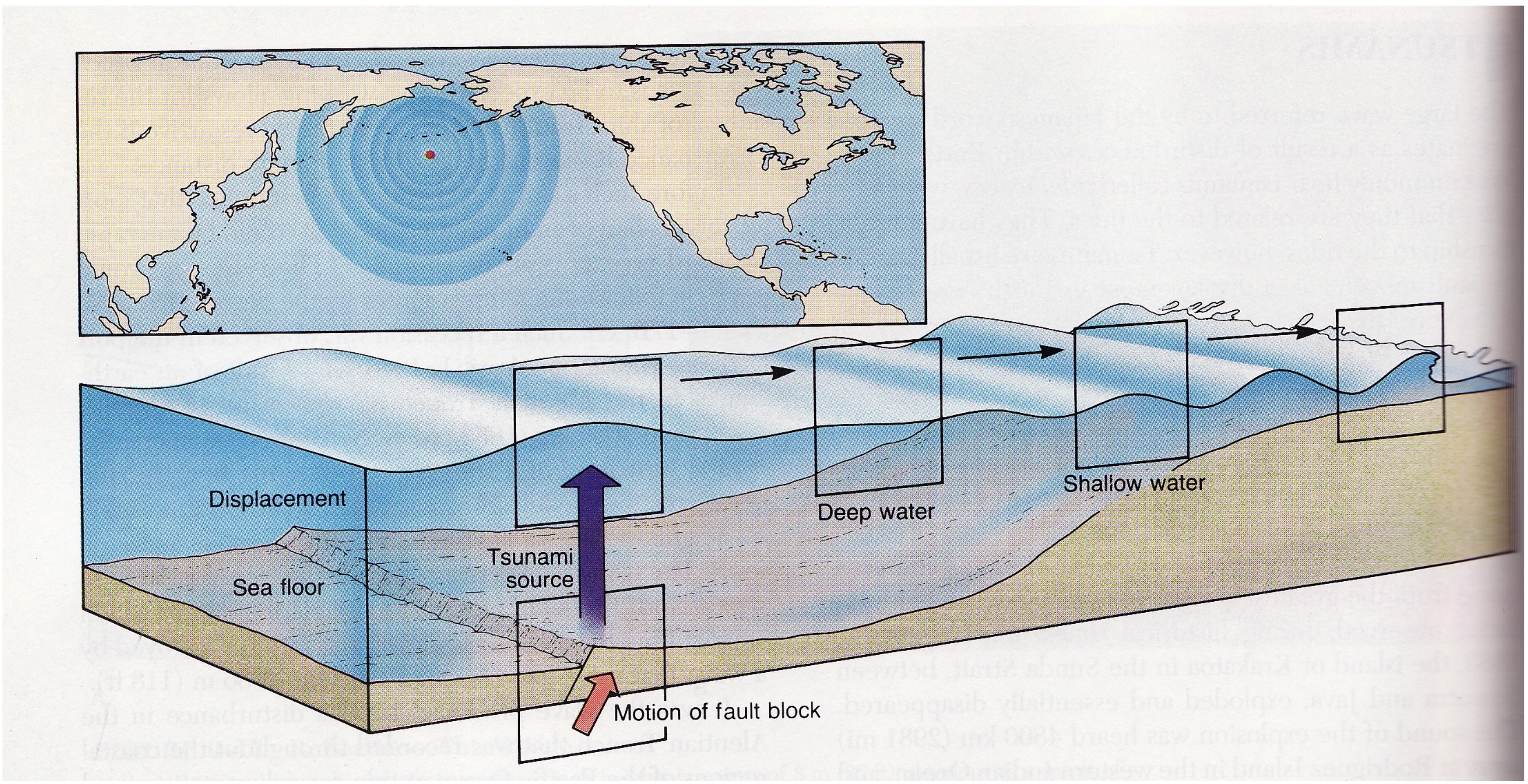
$$c = \sqrt{gH}$$



The Open University, Waves and Tides, 2001.

Tsunami

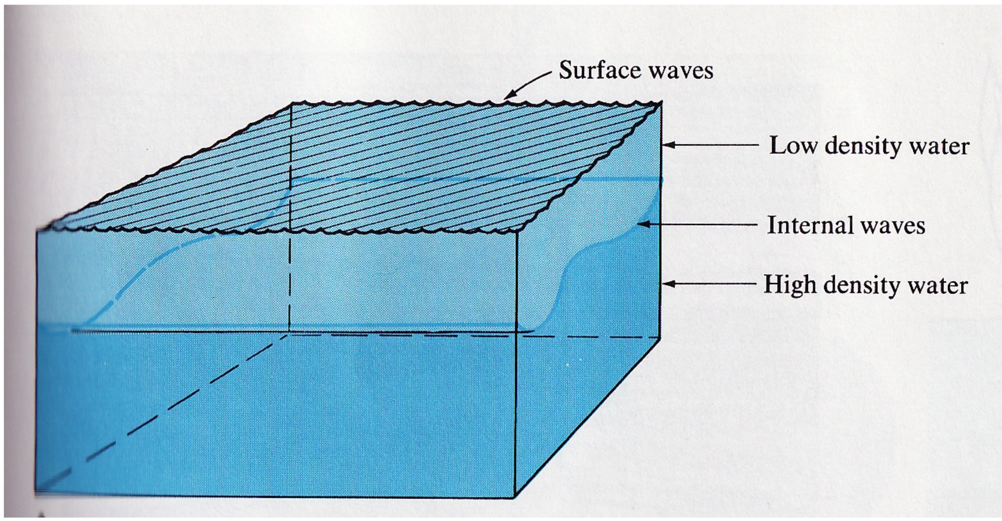

A.

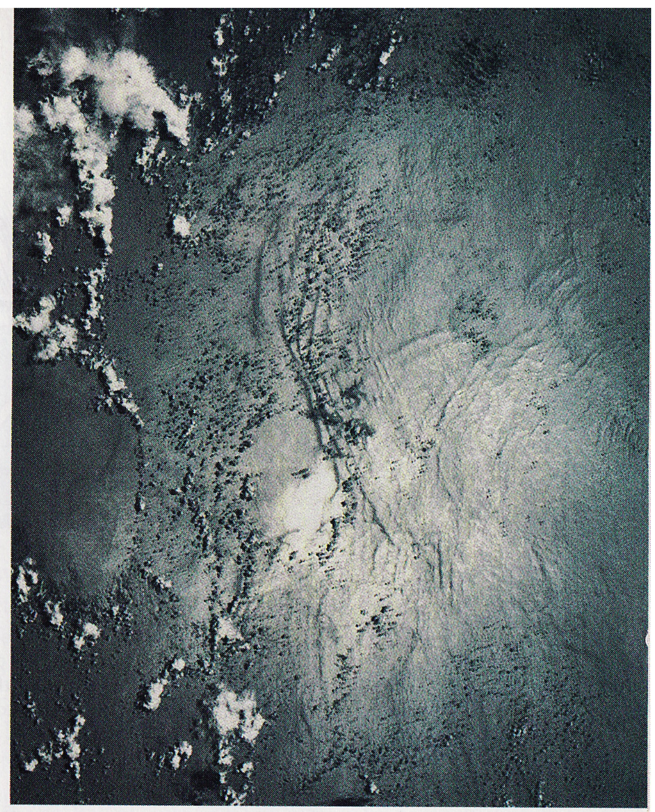

Displacement
Sea floor
Tsunami source
Deep water
Shallow water
Motion of fault block

A Tsunami is essentially a shallow water wave

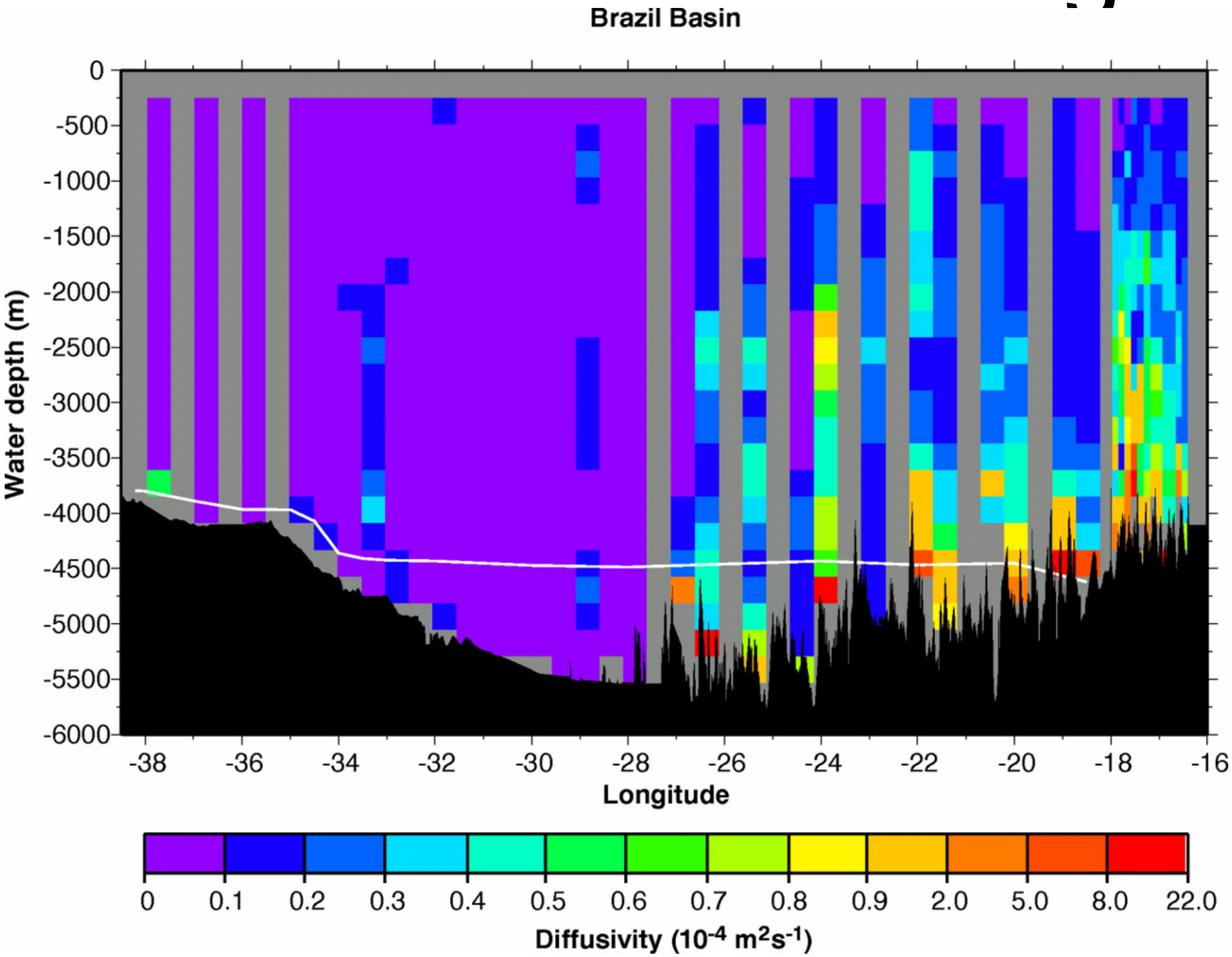
How fast does it propagate?

Internal waves



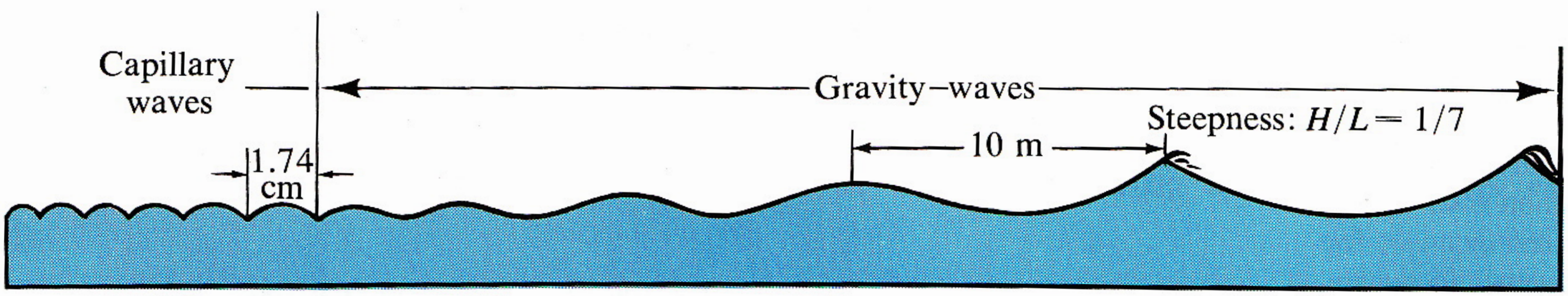


Internal mixing due to waves and wave breaking



Polzin et al. 1997

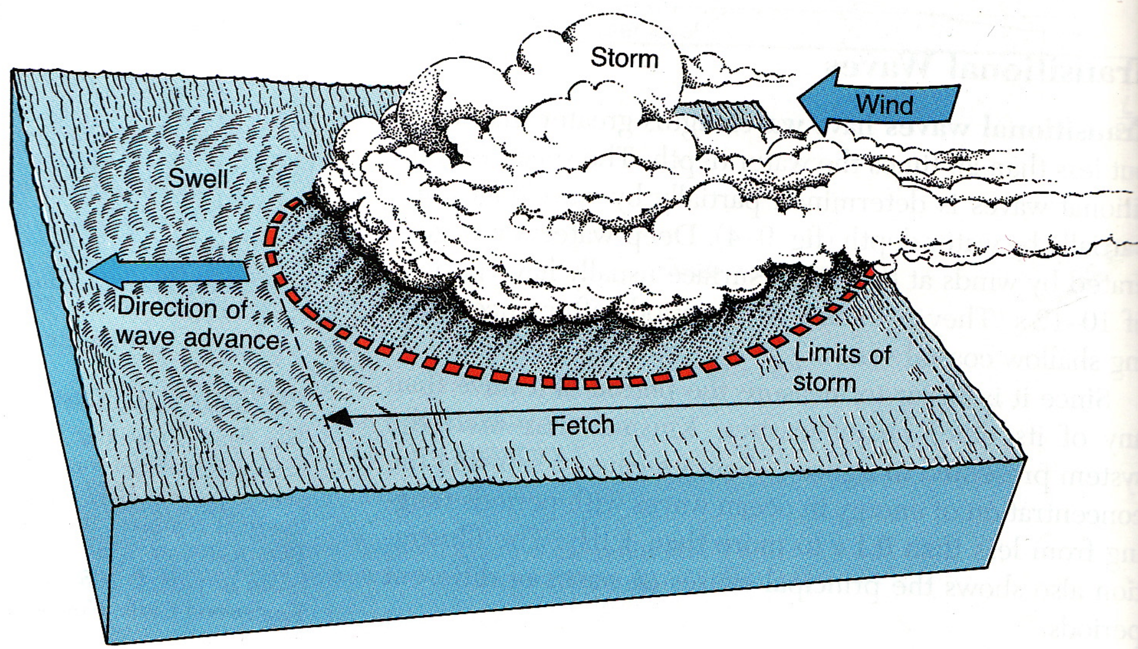
From the sea to the shore

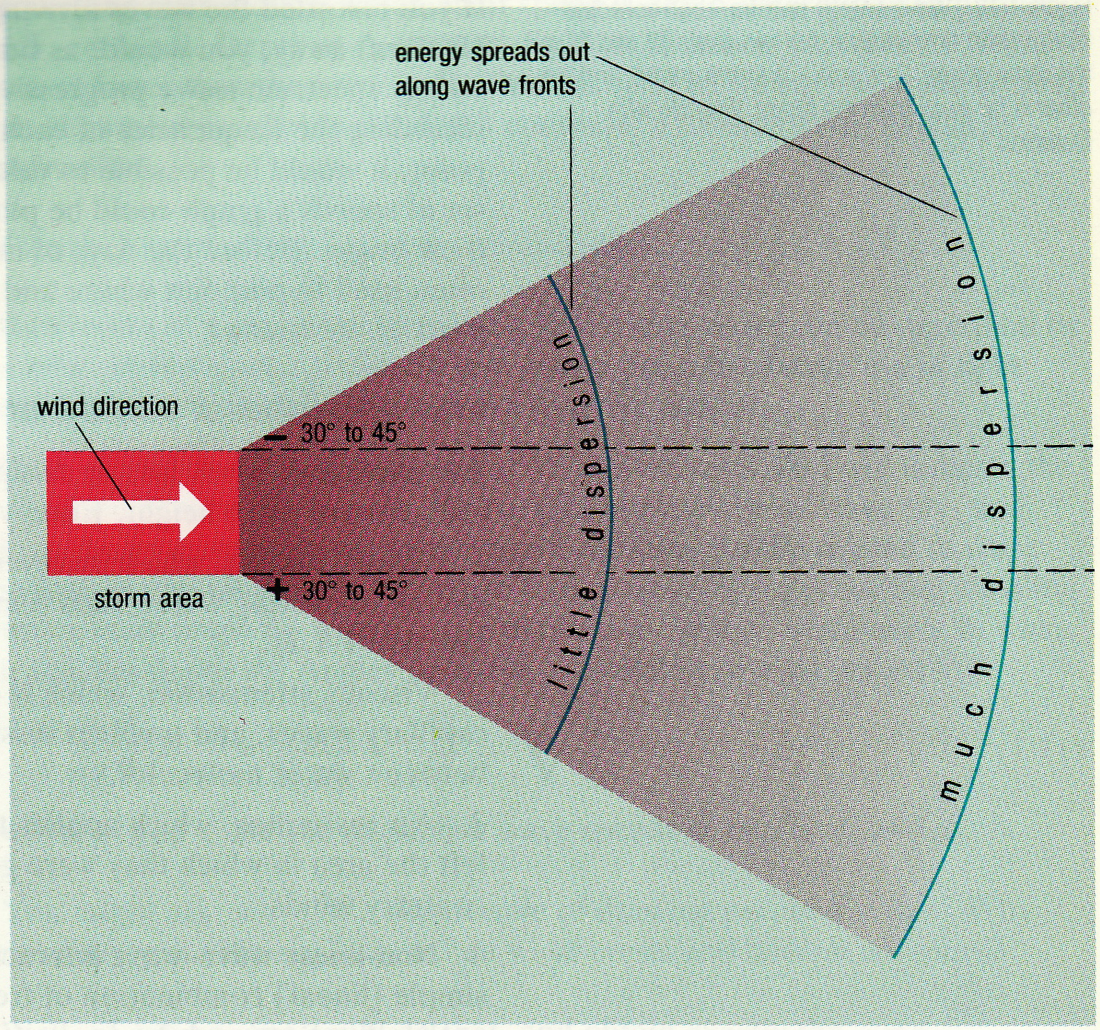

Capillary waves
Gravity–waves
Steepness: H/L = 1/7
10 m
1.74 cm

Swell

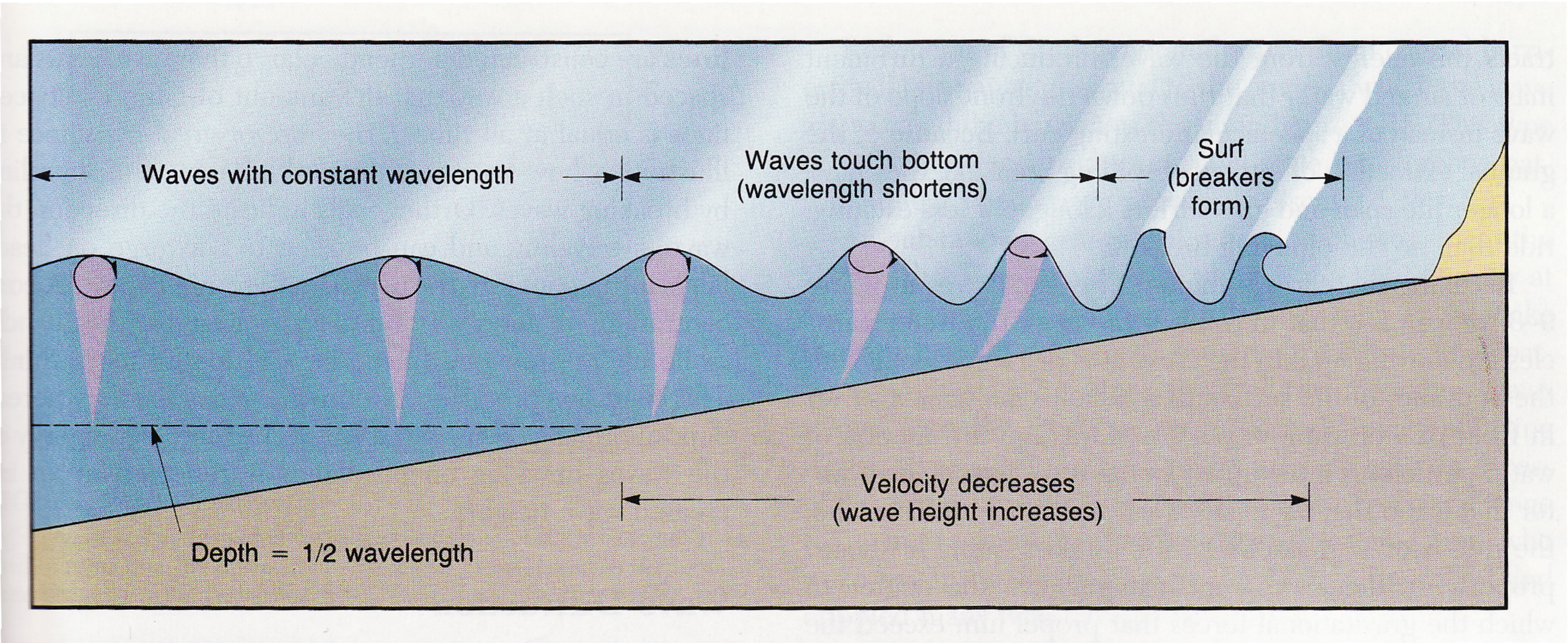
FIGURE 9–7 The Sea and Swell.

As wind blows across the "sea," wave size increases with increased wind speed, fetch, and duration. As waves advance beyond the sea, they continue to advance across the ocean surface as "swell," free waves that are not driven by the wind but sustained by the energy they obtained in the sea.

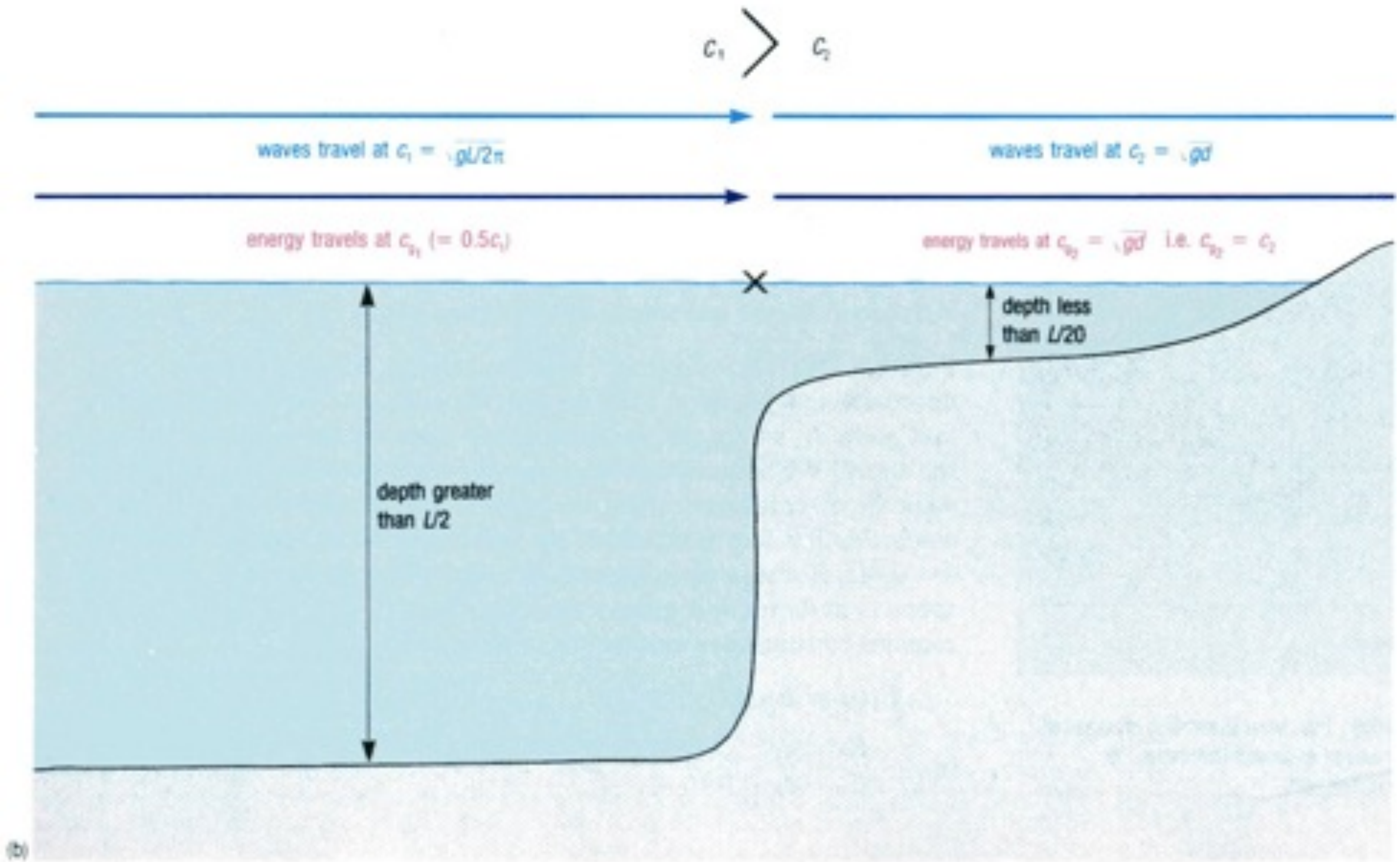



energy spreads out along wave fronts
wind direction
storm area
− 30° to 45°
+ 30° to 45°
little dispersion
much dispersion

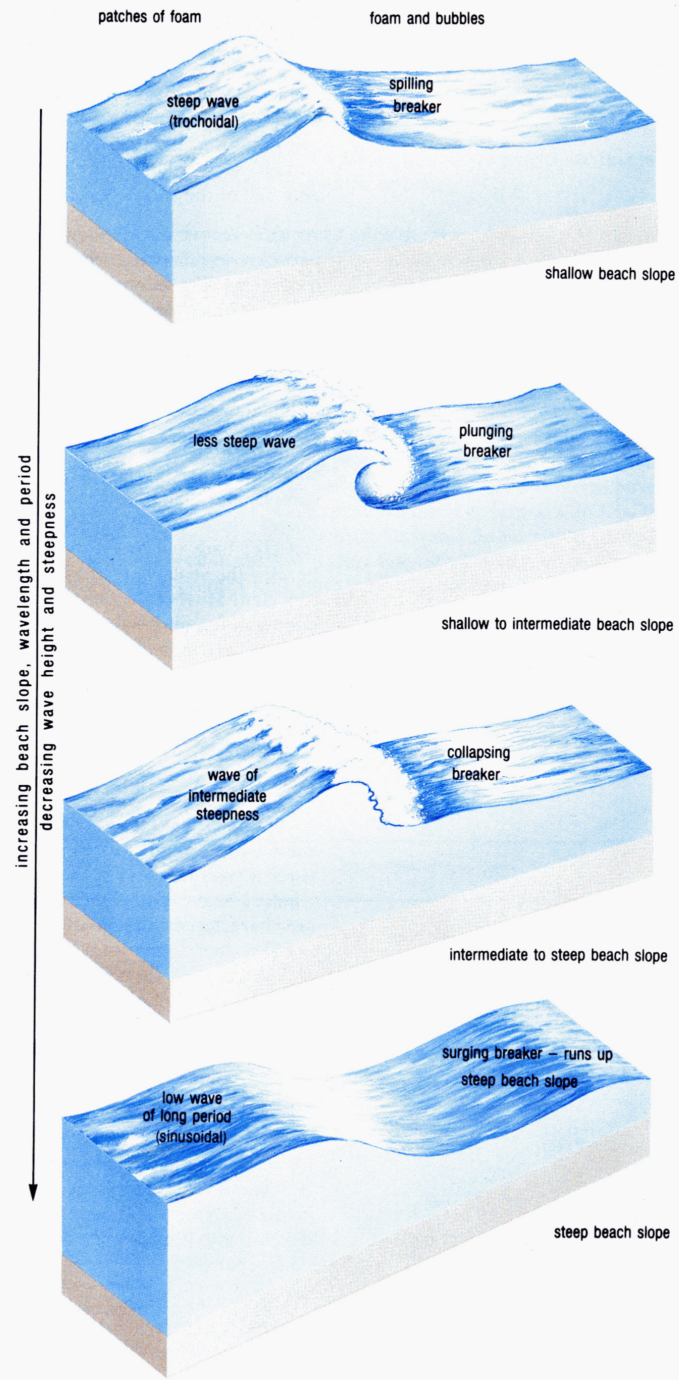
In the surf zone



Energy transformation as wave approaches shore



The Open University, Waves and Tides, 2001.


patches of foam
foam and bubbles
steep wave (trochoidal)
spilling breaker
shallow beach slope
less steep wave
plunging breaker
shallow to intermediate beach slope
wave of intermediate steepness
collapsing breaker
intermediate to steep beach slope
low wave of long period (sinusoidal)
surging breaker – runs up steep beach slope
steep beach slope
increasing beach slope, wavelength and period
decreasing wave height and steepness

gth,

Planetary Waves

Kelvin wave: A kind of surface gravity wave, shallow water

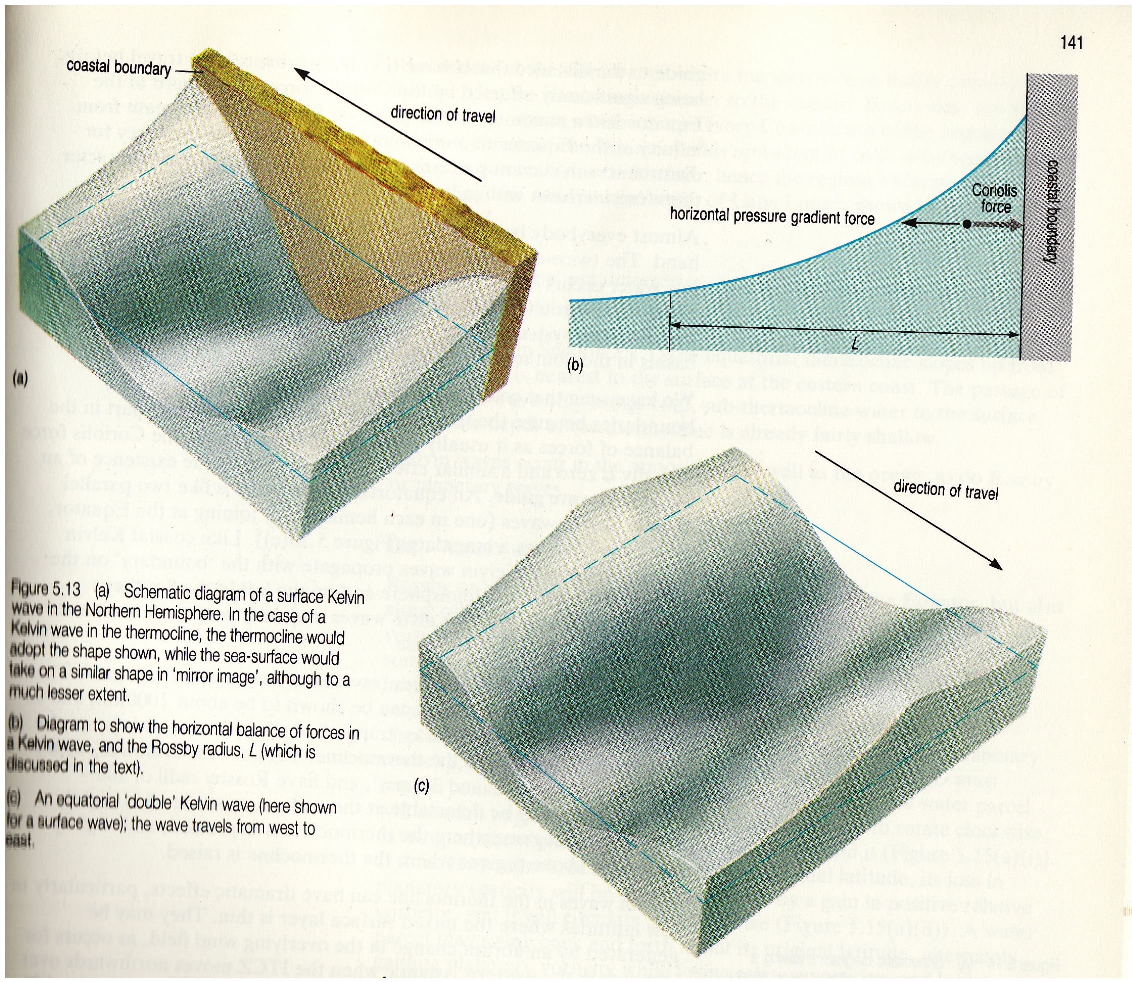
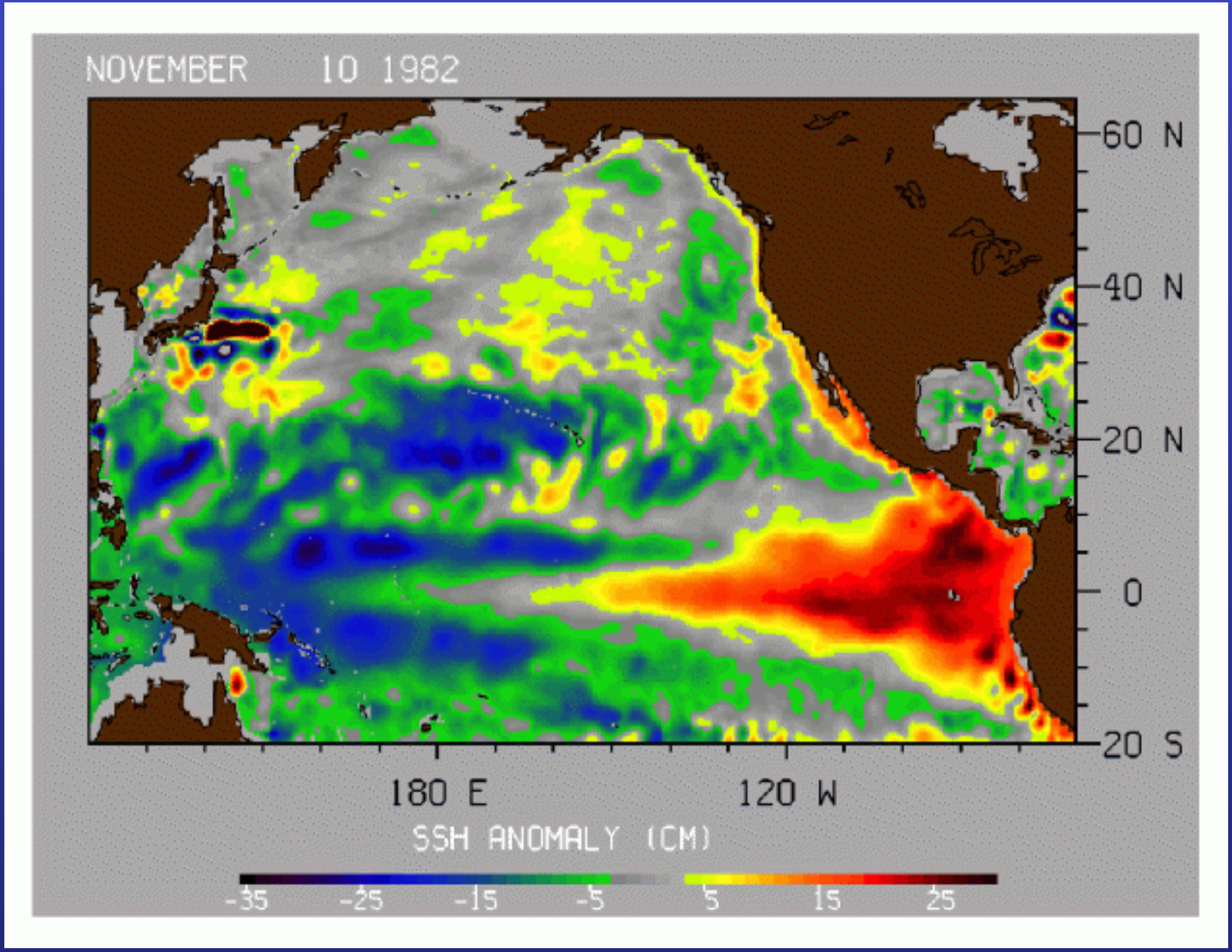


Figure 5.13 (a) Schematic diagram of a surface Kelvin wave in the Northern Hemisphere. In the case of a Kelvin wave in the thermocline, the thermocline would adopt the shape shown, while the sea-surface would take on a similar shape in 'mirror image', although to a much lesser extent.

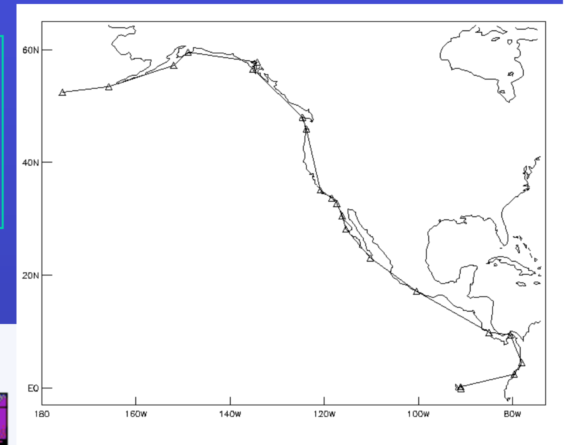
(b) Diagram to show the horizontal balance of forces in a Kelvin wave, and the Rossby radius, L (which is discussed in the text).

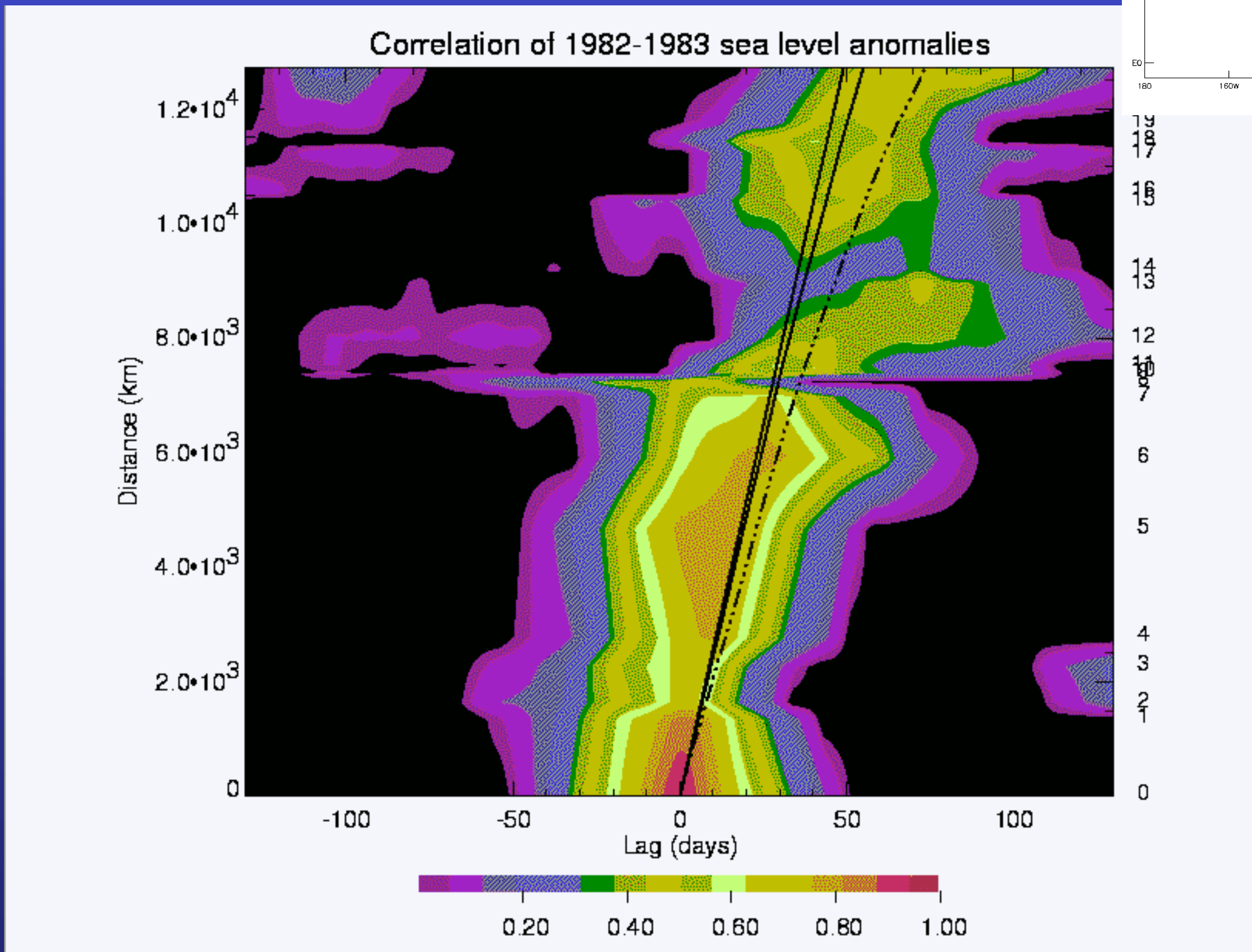
(c) An equatorial 'double' Kelvin wave (here shown for a surface wave); the wave travels from west to east.

Coastal Kelvin Waves



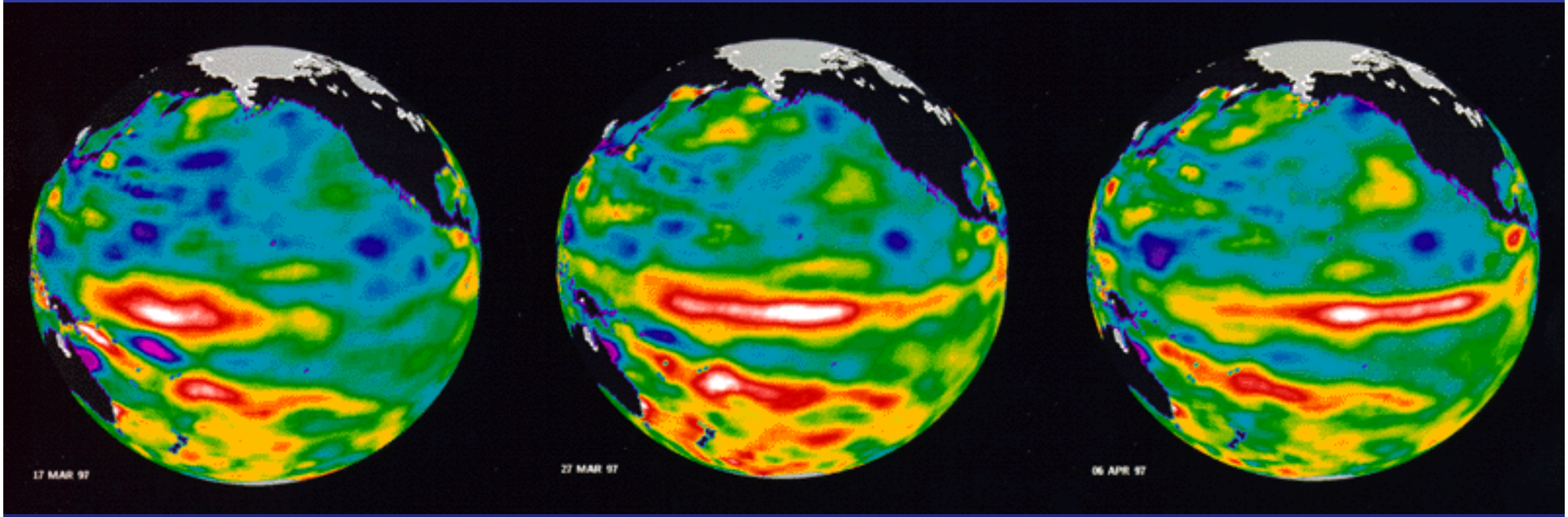
Coastal Kelvin Waves

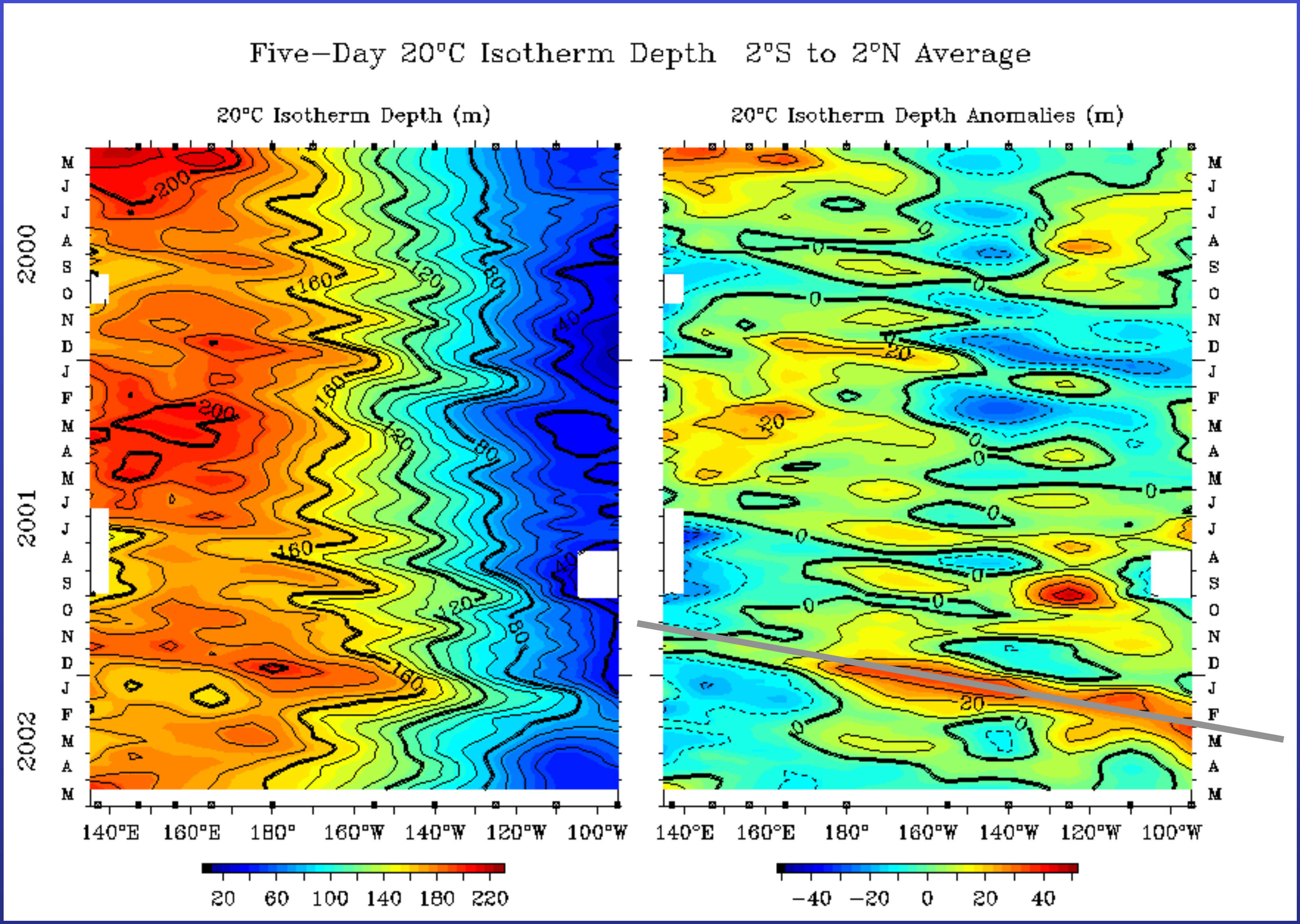




Equatorial Kelvin Waves

- Satellite altimetry from TOPEX/Poseidon
- Scenes are 10 days apart

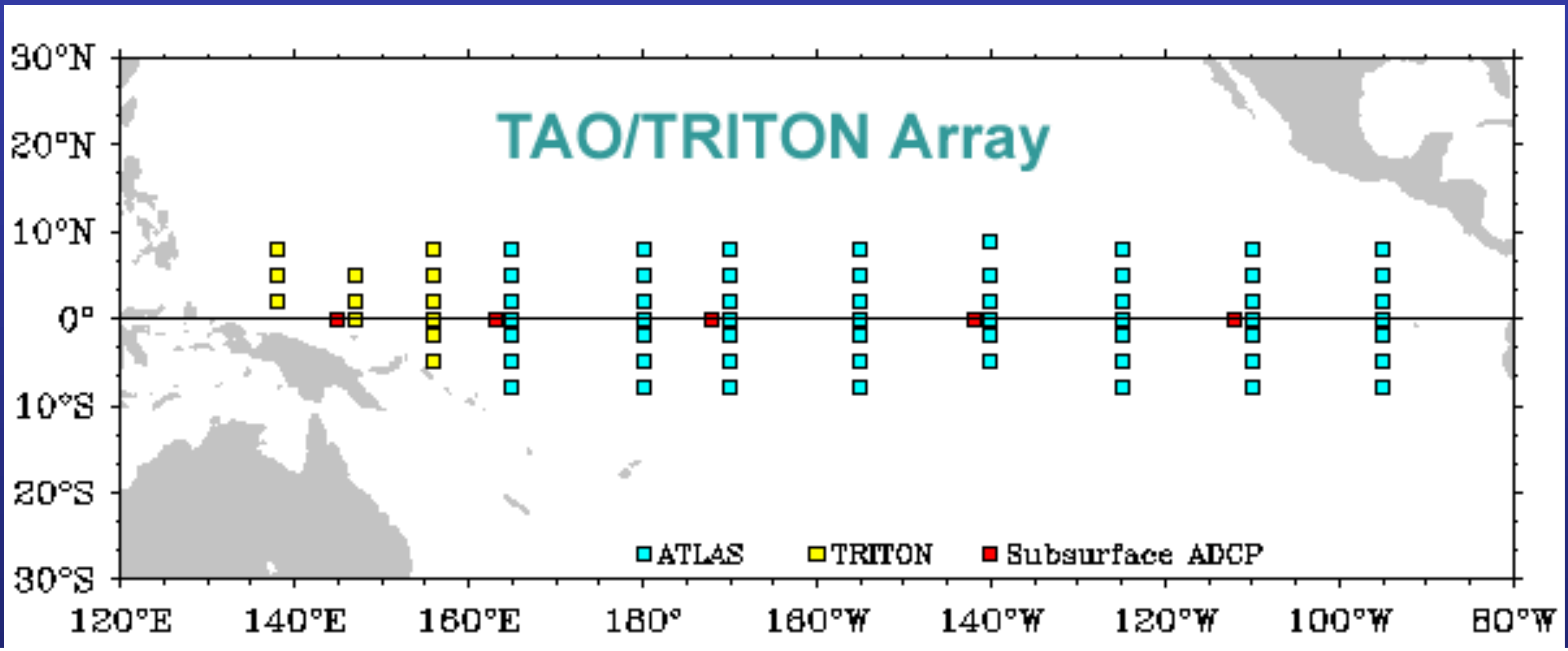




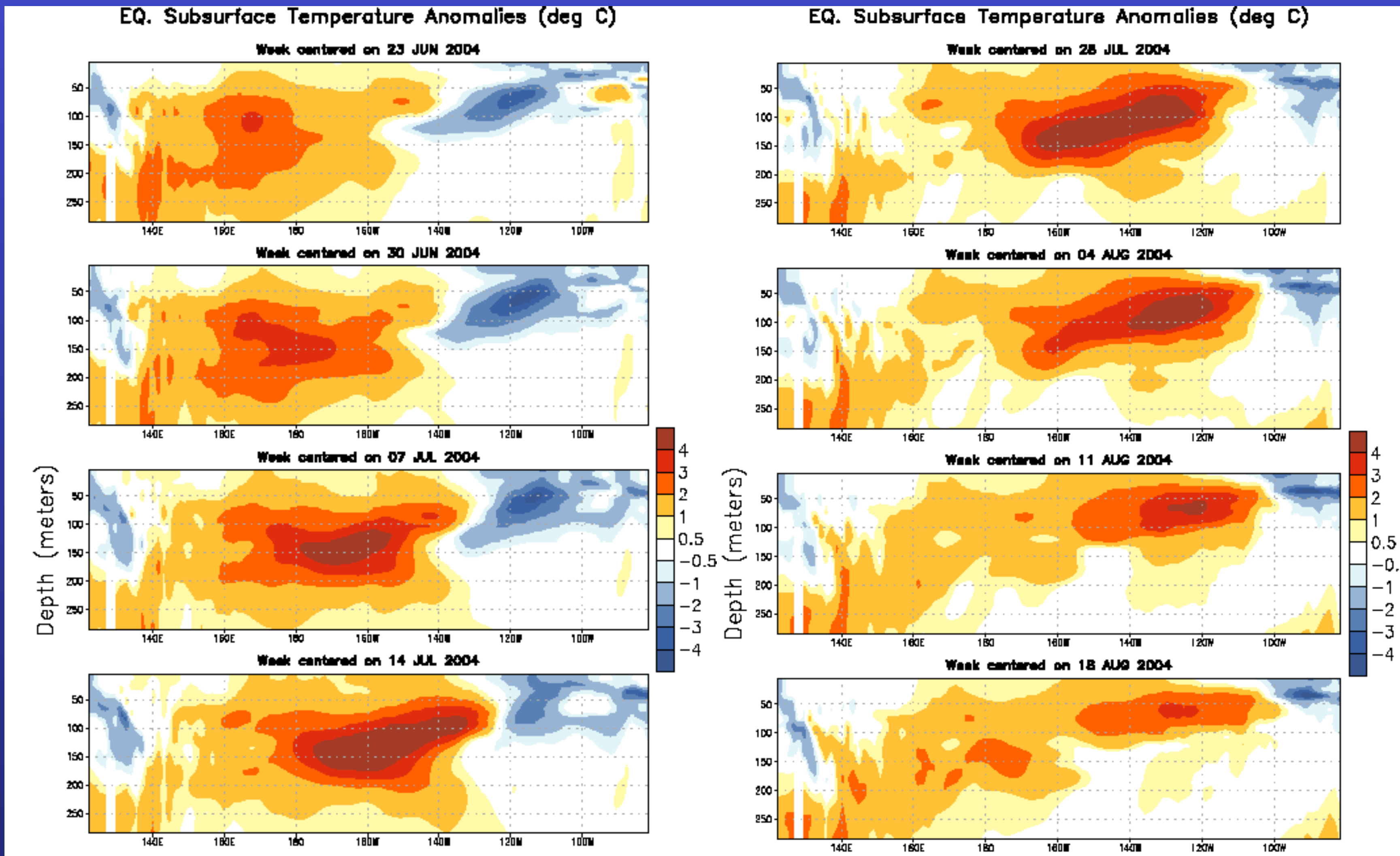
Propagates 13,000 km in 4 months - 1.3 m/s

TOGA-TAO Array

- Equatorial array of buoys
- U.S., Japan & French partnership
- http://www.pmel.noaa.gov/tao/

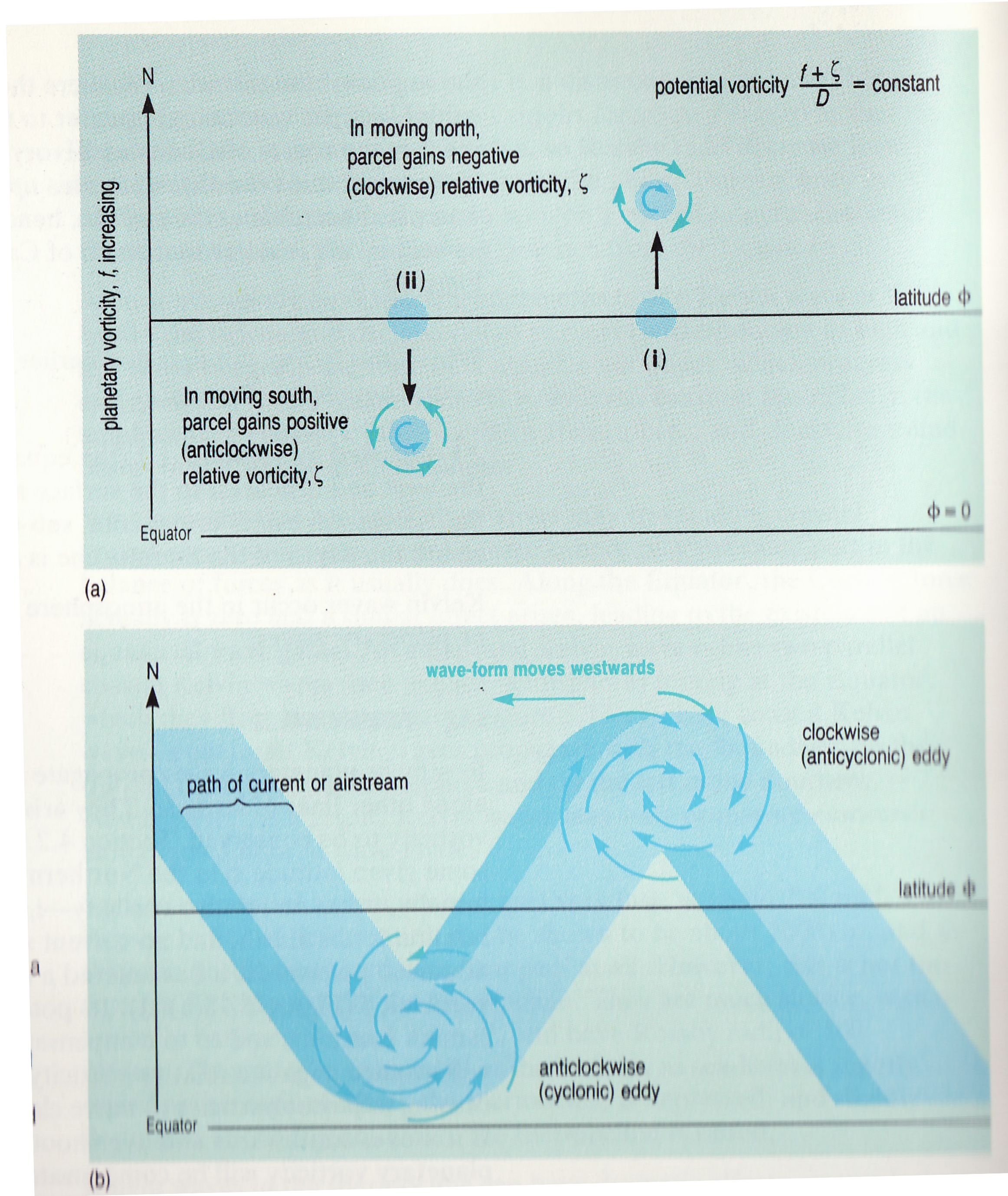


Equatorial Kelvin Waves



Rossby wave

- Restoring force = conservation of PV
- Motion in the horizontal plane



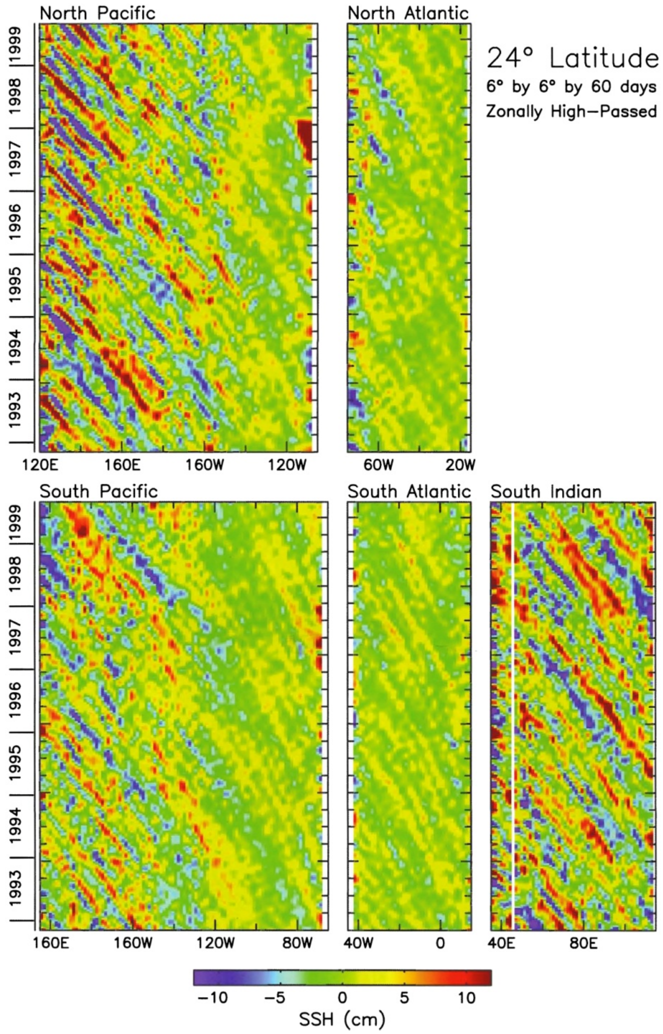


FIGURE 14.18

Surface-height anomalies at 24 degrees latitude in each ocean, from a satellite altimeter. This figure can also be found in the color insert. *Source: From Fu and Chelton (2001).*

TALLEY

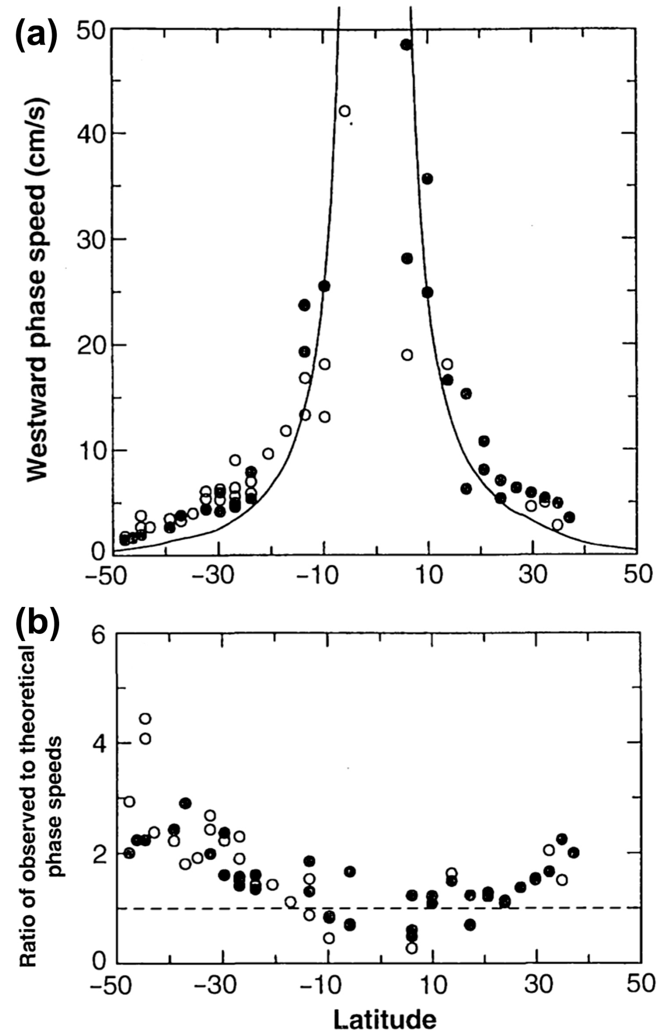
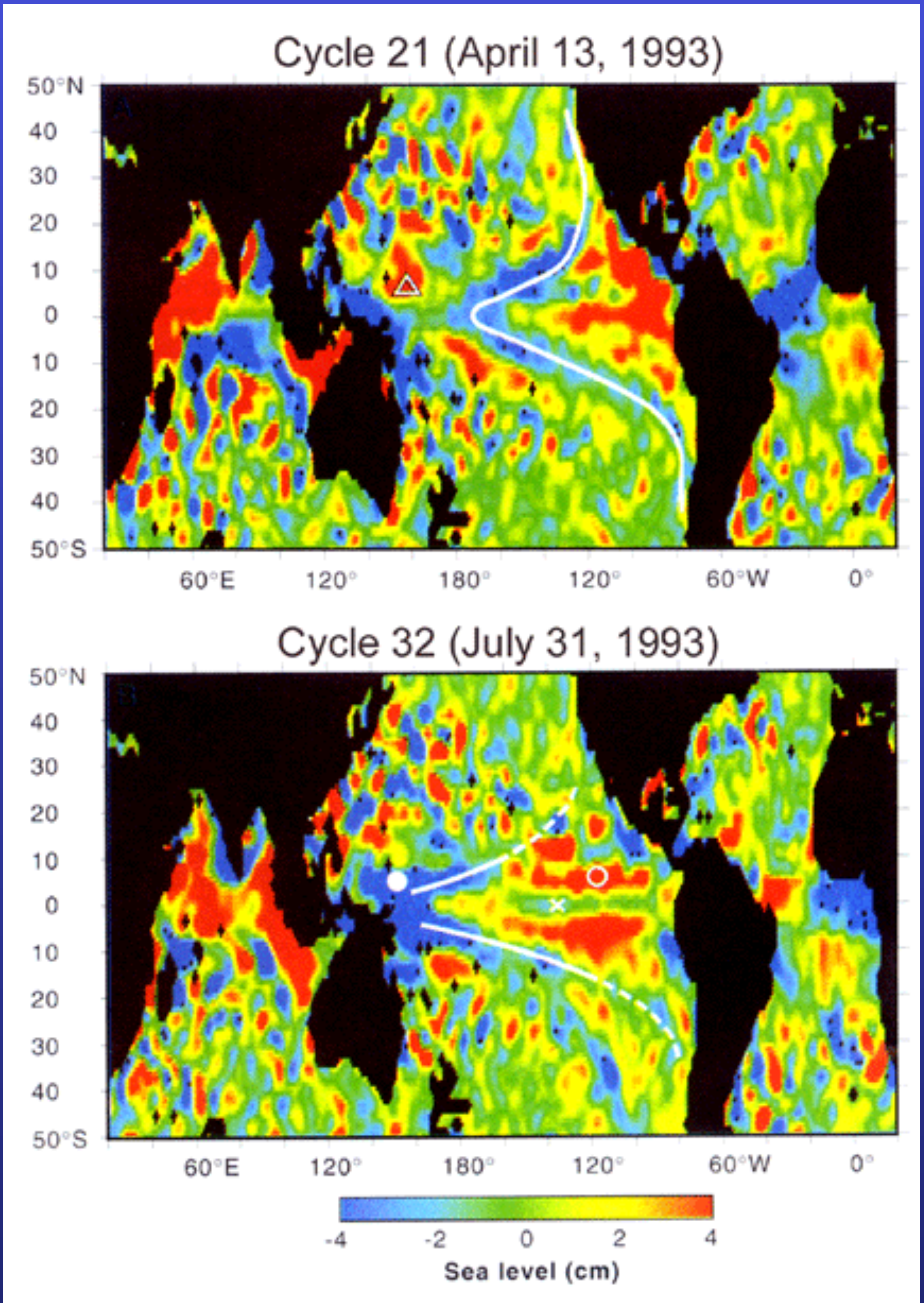


Figure 14.19 (a) Westward phase speeds (cm/sec) in the Pacific Ocean, calculated from the visually most-dominant SSH anomalies from satellite altimetry. The underlying curves are the fastest first-mode baroclinic Rossby waves speeds at each latitude. (b) The ratio of observed and theoretical phase speeds, showing that the observed phase speeds are generally faster than theorized. *Source: From Chelton and Schlax (1996).*

Rossby Waves



Waves critical to ENSO

- Panel 1: Internal equatorial Kelvin wave approaches (on 20C isotherm)
- Panel 2: Coastal Kelvin wave forms
- Panel 3: Reflected Rossby wave and coastal Kelvin