Chapter 10

10.6 Repeat Exercise 10.1 for the stratosphere.

The stratosphere absorbs 3 units of solar radiation, 2 units of outgoing radiation emitted by the Earth's surface, and 6 units of outgoing longwave radiation emitted by the troposphere, for a total of 11 units. It emits a total of 11 units: 6 upward and 5 downward. The upward emission is greater than the downward emission because, on average, temperature increases with height within the stratosphere. Unlike the troposphere, the stratosphere is in radiative equilibrium; i.e., it emits as much radiation as it absorbs.

10.7 Using data in Fig. 10.9 make a rough estimate of the rate of release of sensible and latent heat during autumn and early winter, when the ocean mixed later is cooling and deepening.

Consider the top-most 100-m layer cools from a vertically averaged temperature of ∼18°C in September to ∼15°C in January, a drop of ∼3°C in ∼100 days. Hence, the rate of energy loss through the surface is

$$F = \frac{\rho_w c_w H \delta T}{\delta t}$$

where... Substituting numerical values, we obtain

$$\begin{aligned} &\sim \quad \frac{10^3 \times 4218 \times 100 \times 3^\circ}{100 \text{ days} \times 0.864 \times 10^5 \text{ s day}^{-1}} \\ &\sim \quad 150 \text{ W m}^{-2} \end{aligned}$$

10.8 If the time series of a climatic variable is perfectly sinusoidal with amplitude A, prove that its root mean squared amplitude or standard deviation is $A\sqrt{2}$.

The variance of a sine wave with amplitude A is

$$\begin{aligned} Var &= \frac{1}{2\pi}\int_0^{2\pi} A\cos^2\phi d\phi \\ &= \frac{A}{2\pi}\int_0^{2\pi} \frac{1+\cos 2\phi}{2} d\phi \\ &= \frac{A}{2\pi} \times \pi \\ &= A/2 \end{aligned}$$

Hence, the root mean squared amplitude or standard deviation is $A\sqrt{2}$.

10.10 The standard deviation of monthly mean temperature at a certain station averaged over the winter months December-March is 5.0° C. The standard deviation of winter seasonal mean temperature is 3.0°C. What is the standard deviation of monthly mean temperature about the respective seasonal means for individual winters?

Monthly mean temperature may be expressed as

$$T = T_m + T'_w + T$$

Answer 4.0°C

10.11 Prove that

$$\overline{x'y'} = \overline{xy} - \overline{x}\,\overline{y}$$

[**Hint**: Make use of the fact that $\overline{\overline{x}y'} = \overline{x'\overline{y}} = 0$.]

10.12 Suppose that the Earth warms by 3°C during the 21st century and that the entire ocean were to warm by the same amount. Assume that the cryosphere remains unchanged. In the global energy balance in Fig. 10.1, by how many W m^{-2} would the net incoming solar radiation have to exceed the outgoing Earth radiation at the top of the atmosphere?

The net downward flux of energy from the ocean surface is

$$F = \frac{mc_w \delta T}{\delta t}$$

where m is the mass per unit area of the oceans... Substituting values (using the value given in Table 2.2 for m) yields

$$\begin{aligned} F &= \frac{\left(2700 \times 10^3\right) \text{ kg} \times 4218 \text{ J kg}^{-1} \text{ K}^{-1} \times 3 \text{ K}}{100 \times 365 \times 0.864 \times 10^5 \text{ s}} \\ &= 10.8 \text{ W m}^{-2} \end{aligned}$$

10.13 On the basis of the global energy balance in Fig. 10.1, estimate what the surface temperature of the Earth would be in the absence of latent and sensible heat fluxes. Assume that the fraction of the longwave radiation emitted from the Earth's surface that is absorbed by the atmosphere and re-emitted back to the surface remains unchanged.

In the present climate the Earth emits 110 units of energy in the form of longwave radiation and it receives 89 units of longwave radiation from the atmosphere. Hence the net emission is 21 units. In the absence of latent and sensible heat fluxes the net upward flux from the Earth's surface would need to be equal to the absorbed solar radiation, which is 50 units. If the ratio of downward to upward emission remains the same as it is in today's climate, the total emission from the Earth's surface would need to be equal to $50 \times 110/21 = 262$ units, where each unit is equivalent to 3.45 W m^{-2}. Hence, the equivalent blackbody temperature would be

$$T_E = \left(\frac{262 \times 3.45}{5.67 \times 10^{-8}}\right)^{1/4} = 355 \text{ K}$$

10.14 Using data in Fig. 10.2, estimate the poleward flux of energy by the atmosphere and oceans across 38°N, where the incoming and outgoing radiation curves intersect.

The excess solar energy absorbed in the $0° - 38°$ latitude belt must be transported poleward across 38°N. The energy transported across 38°N must be equal to this net flux times the area of the Earth's surface that lies between the equator and 38°N; that is,

$$\text{Transport} = \overline{F} \times \; 2\pi R_E^2 \int_{0°}^{38°} \cos\phi d\phi$$

where F is the downward net radiation over the area of the northern hemisphere equatorward of 38°N and R_E is the radius of the Earth. From Fig. 10.2 it is evident that $F \sim 40$ W m^{-2}. The area equatorward of 38°N is

$$\begin{aligned} 2\pi R_E^2 \int_{0°}^{38°} \cos\phi d\phi &= 2\pi R_E^2 \sin 38° \\ &= 2\pi \times \left(6.37 \times 10^6\right)^2 \times 0.616 \\ &= 1.56 \times 10^{14} \end{aligned}$$

Hence, the estimated poleward transport is 6.24×10^{15} W.

10.15 Compare the daily insolation upon the top of the atmosphere (a) at the North Pole at the time of the summer solstice and (b) at the equator at the time of the equinox. The *solar declination angle* (the astronomical analog of geographic latitude; i.e., the latitude at which the sun is directly overhead at noontime) at the time of the summer solstice is 23.45° and the Earth-Sun distances are 1.52 and 1.50×10^8 km, respectively.

Answer (a) 46.4 versus 38.0 MJ m^{-2} day^{-1}(see also Fig. 10.5)

10.16 Consider the response of the Earth's equivalent blackbody temperature T_E to a volcanic eruption that increases the planetary albedo, resulting in a radiative forcing at the top of the atmosphere of $\delta F = -\,2$ W m^{-2}. (a) Calculate the equilibrium response. (b) Suppose that the atmosphere is well mixed and thermally isolated from the other components of the Earth system; that the increase in planetary albedo is instantaneous; and that the albedo remains constant at the higher value after the eruption. Show that T_E drops toward its new equilibrium value exponentially, approaching it with an e-folding time equal to the atmospheric radiative relaxation time defined and estimated in Exercise 4.29. [**Hint**: Consider the energy balance at the top of the atmosphere. Make use of the fact that $\delta F/F << 1$ and $\delta T_E/T_E << 1$.]

(a) Making use of the results of Exercise 4.21, the equivalent blackbody temperature would be reduced by the factor

$$\frac{\delta T_E}{T_E} = \frac{1}{4}\frac{\delta F}{F}$$

Substituting values, we obtain

$$\begin{aligned} \delta T_E &= \frac{1}{4} \times \frac{2}{239} \times 255 \text{ K} \\ &= 0.53 \text{ K} \end{aligned}$$

(b) Initially the atmosphere will cool at a rate

$$\begin{aligned} c_p \frac{p_s}{g}\frac{dT}{dt} &= \delta F = \delta \left(\sigma T^4\right) \\ &= 4\sigma T^3 \delta T_E \end{aligned}$$

where p_s is the surface pressure. Solving for dT/dt, we obtain

$$\frac{dT}{dt} = \gamma \delta T_E$$

where

$$\gamma = \frac{4\sigma T^3}{c_p p_s g^{-1}}$$

as in Exercise 4.29. Hence, the atmosphere will cool exponentially toward the new equilibrium temperature $T_E + \delta T_E$. Substituting values $T = 255$ K, $p_s = 10^5$ Pa, $c_p = 1004$, $g = 9.8$, we obtain a value of $\gamma = 3.67 \times 10^{-7}$ s^{-1}, a radiative relaxation time $1/\gamma$ of 2.72×10^6 s (or 31.5 days), and $dT/dt = 1.95 \times 10^{-7}$ K s^{-1} or 0.0186 K day^{-1}.

10.17 Rework Exercise 10.16, but assuming that the radiative flux at the top of the atmosphere abruptly returns to its preeruption value exactly a year after the eruption. Estimate the drop in T_E during the year after the eruption (a) assuming that the atmosphere is well mixed and thermally isolated from the other components of the Earth system and (b) assuming that the atmosphere remains in thermal equilibrium with a 50-m-deep ocean mixed layer that covers the planet. (c) Under which of these scenarios does the Earth system lose more energy during the year that the volcanic debris are present in the atmosphere? (d) Which of these scenarios is more realistic, and why?

In this case

$$\gamma = \frac{4\sigma T^3}{\rho_w c_w D}$$

where ρ_w and c_w are the density and specific heat of sea water, respectively and D is the depth of the mixed layer. Substituting values $\rho_w = 10^3$, $c_w = 4218$, and $D = 50$ m, we obtain $\gamma = 1.79 \times 10^{-8}$, a radiative relaxation time $1/\gamma$ of 5.59×10^{-7}s (or 1.8 year), and $dT/dt = 9.5 \times 10^{-9}$ K s^{-1}.

(c) The Earth system loses more energy in the scenario in Exercise 10.17 because the Earth's equivalent blackbody temperature takes about 20 times as long to equilibrate with the decreased albedo due to the volcanic eruption.

(d) The scenario in this exercise is much more realistic.

10.18 Which of the following has the largest impact on the Earth's equivalent blackbody temperature? (a) The ˜0.07% variation sun's emission that is observed to occur in association with the 11-year sunspot cycle. (b) The flux of geothermal energy from the interior of the Earth (0.05 W m^{-2}). (c) The consumption of energy by human activities (10^{13} W).

Answer .

10.19 Two feedback processes capable of amplifying greenhouse warming by factors of 1.5 and 2.0, respectively, if each were acting in isolation, would be capable of amplifying it by a factor of 6 if they were acting in concert.

For a single feedback acting in isolation

$$g = \frac{1}{1-f}$$

Hence

$$f = \frac{g-1}{g}$$

Substituting values we obtain feedback factors of 0.333 and 0.500. When forcings are combined, the feedback factors are additive, i.e.,

$$g = \frac{1}{1-\sum f}$$

Hence, when the forcings are combined, the gain is

$$g = \frac{1}{1-0.833} = \frac{1}{0.167} = 6$$

10.20 (a) Without using Eq. (10.9) show that for a case of a single auxiliary variable y, the change in global mean surface air temperature T_s resulting from an incremental climate forcing δF is given by

$$\delta T_s = \lambda_0 \delta F \left(1 + f + f^2 + f^3 ......\right) \qquad (10.16)$$

where f is the feedback factor as defined in Eq. (10.7). (b) Show that (10.9) follows directly from (10.16). (c) Give a verbal definition of *feedback factor* based on this exercise.

In the absence of feedbacks, T_s would rise by the increment $(\delta T_s)_0 = \lambda_0 \delta F$. As a result of this temperature rise $(\delta T_s)_0$, the feedback would cause T_s to rise by an additional increment $(\delta T_s)_1 = f(\delta T_s)_0 = f\lambda_0\delta F$. As a result of this secondary temperature rise $(\delta T_s)_1$, the feedback would cause T_s to rise by an additional increment $(\delta T_s)_2 = f(\delta T_s)_1 = f^2\lambda_0\delta F$, and so on. The total temperature rise is

$$\begin{aligned}\delta T_s &= (\delta T_s)_0 + (\delta T_s)_1 + (\delta T_s)_2 + ... \\ &= \lambda_0 \delta F \left(1 + f + f^2 + f^3......\right)\end{aligned}$$

Summing over the series, we obtain

$$\delta T_s = \frac{\lambda_0 \delta F}{1 - f}$$

which is equivalent to (10.9) in the text.

10.25 See General web page

Solutions to the remaining exercises in Chapter 10 will be provided as time permits.