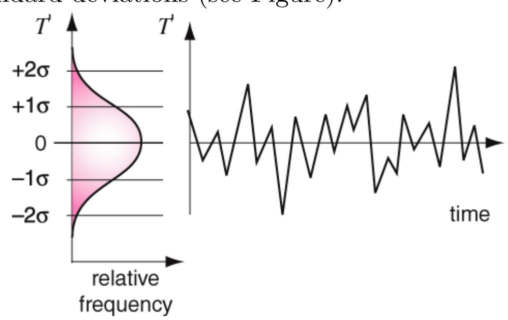
Chapter 9

9.8 Estimate the temperature variance for the velocity trace at the bottom of Fig. 9.6.

SOLUTION If turbulent fluctuations are approximately normally distributed, then about 68% of the T' values are within plus/minus 1 standard deviation of the mean (where the mean = 0 for T'), and 95% are within plus/minus 2 standard deviations (see Figure).



By eye, we can estimate the mean value (corresponding to $T' = 0$) in the bottom trace of Fig 9.6. Given the 1°C temperature scale near the top center of the figure, by eye it appears that 95% of all the T' values are within plus/minus 0.9°C.

If $2{\cdot}\sigma_T = 0.9$ °C, then $\sigma_T = 0.45$ °C and $\sigma_T^2 = 0.2$ °C^2 = temperature variance. However, this is not precise. By eye, you might have found $2{\cdot}\sigma_T = 0.8$, which gives $\sigma_T^2 = 0.16$ °C^2. Thus, there is a range of acceptable values for this answer.

9.9 Prove that the definition of covariance reduces to the definition of variance for the covariance between any variable and itself.

SOLUTION $\text{cov}\,(w, \theta) = \overline{w'\theta'}$ from Eq. (9.5). Let $w = A$ and $\theta = A$, where A is any variable. Then $\text{cov}\,(A, A) = \overline{A'A'} = \overline{A'^2}$ (identify this as Eq. 9.3a).

But $\sigma_u^2 = \overline{[u'^2]}$ from eqn. (9.4). Let $u = A$. Thus: $\sigma_A^2 = \overline{[A'^2]}$ (eqn. 9.3b). Equating equations 9.3a and b gives $\text{cov}\,(A, A) = \sigma_A^2$.

9.10 Given the following variances in m^2 s^{-2}.

Where:	**Location A**		**Location B**	
When (UTC):	**1000**	**1100**	**1000**	**1100**
σ_u^2	0.50	0.50	0.70	0.50
σ_v^2	0.25	0.50	0.25	0.25
σ_w^2	0.70	0.50	0.70	0.25

When, where, and for which variables is the turbulence: (a) stationary, (b) isotropic, and (c) homogeneous?

SOLUTION a) Stationary: The variances that are not changing with time are: σ_u^2 at A, and σ_v^2 at B.

b) Isotropic: The variances that are the same in all directions are: Location A at 11 UTC.

c) Homogeneous: Variances that are the same at different locations are:

σ_v^2 at 10 UTC, σ_w^2 at 10 UTC, and σ_u^2 at 11 UTC.

9.11 Given the following synchronized time series for T(°C) and w(m s^{-1}), find: a) mean temperature, b) mean velocity, c) temperature variance, d) velocity variance, e) kinematic heat flux

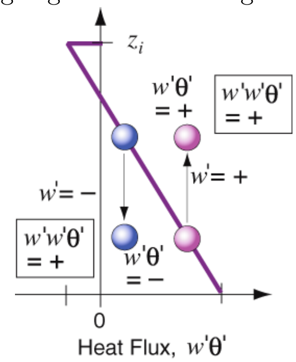
T	21	22	20	25	25	15	18	23	21	24	16	12	19	22
w	1	-2	0	-3	2	-2	-3	3	0	0	1	4	-2	-3

SOLUTION

	T (°C)	w (m/s)	T'	w'	T'^2	w'^2	$w'T'$
	21	1	0.786	1.286	0.61735	1.65306	1.010204
	22	-2	1.786	-1.714	3.18878	2.93878	-3.06122
	20	0	-0.214	0.286	0.04592	0.08163	-0.06122
	25	-3	4.786	-2.714	22.9031	7.36735	-12.9898
	25	2	4.786	2.286	22.9031	5.22449	10.93878
	15	-2	-5.214	-1.714	27.1888	2.93878	8.938776
	18	-3	-2.214	02.714	4.90306	7.36735	6.010204
	23	3	2.786	3.286	7.7602	10.7959	9.153061
	21	0	0.786	0.286	0.61735	0.08163	0.22449
	24	0	3.786	0.286	14.3316	0.08163	1.081633
	16	1	-4.214	1.286	17.7602	1.65306	-5.41837
	12	4	-8.214	4.286	67.4745	18.3673	035.2041
	19	-2	-1.214	-1.714	1.47449	2.93878	2.081633
	22	-3	1.786	-2.714	3.18878	7.36735	-4.84694
Avg =	**20.21**	**-0.286**	0.000	0.000	**13.9**	**4.92**	**-1.582**
	(a) $\overline{T}$	(b) $\overline{w}$			(c) $\overline{T'^2}$	(d) $\overline{w'^2}$	(e) $\overline{w'T'}$

9.16 Given the heat-flux profile of Fig. 9.22, extend the method of Fig. 9.8 to estimate the sign of the triple correlation $\overline{w'w'\theta'}$ in the mid-mixed layer, which is one of the unknowns in Eq. (9.11).

SOLUTION Combining Figs. 9.8 and 9.22 gives



Both the up-moving and down-moving portions of a turbulent eddy contribute to positive $w'w'\theta'$. Thus the average of both up and down gives $\overline{w'w'\theta'}$ = positive. This generic method can be used with many other turbulent terms, but give a reasonable estimate only if no large eddies are present.

9.17 Given: $F_H = 0.2$ K m s^{-1}, $z_i = 1$ km, $u_* = 0.2$ m s^{-1}, $T = 300$ K, $z_0 = 0.01$ m, find and explain the significance of the values of: a) Deardorff velocity scale, b) Obukhov length, c) convective time scale, d) wind speed at z =30 m.

SOLUTION

F_H (K m/s)=	0.2	(a)	w_* (m/s) =	1.87 m s^{-1}
z_i (m) =	1000	(b)	L (m) =	-3.06 m
u_* (m/s) =	0.2	(c)	t_* (min) =	8.9 min
T (K) =	300	(d)	V at 30 m (m/s) =	4 m s^{-1}
z_0 (m) =	0.01			

a) Use Eq. (9.13), and assume $T_v = T$:

$$w_* = \left[\frac{\left(9.8 \text{ m s}^{-2}\right) \cdot (1000 \text{ m}) \cdot \left(0.2 \text{ K}\cdot\text{ms}^{-2}\right)}{300 \text{ K}}\right]^{1/3} = 1.87 \text{ m s}^{-1}$$

which is a typical vertical velocity in a thermal.

b) Use Eq. (9.15):

$$L = \frac{-\left(0.2 \text{ ms}^{-1}\right)^3 \cdot (300 \text{ K})}{(0.4) \cdot (9.8 \text{ m s}^{-2}) \cdot \left(0.2 \text{ K}\cdot\text{m s}^{-2}\right)} = -3.06 \text{ m}$$

Below height 3.06 m, mechanical production of turbulence exceeds buoyant production.

c) Use the first Eq. (9.16)

$$t_* = \frac{1000 \text{ m}}{1.87 \text{ m s}^{-1}} = 535 \text{ s} = 8.9 \text{ min}$$

This is the turnover time for a thermal circulation.

d) Use Eq. (9.22):

$$v = \left(\frac{0.2 \text{ m s}^{-1}}{0.4}\right) \ln\left(\frac{30 \text{ m}}{0.01 \text{ m}}\right) = 4 \text{ m s}^{-1}$$

Wind speed increases logarithmically with height.

9.19 If the wind speed is 5 m s^{-1} at $z = 10$ m, and the air temperature is 20°C at $z = 2$ m, then: (a) What is the value of sensible heat flux at the surface of unirrigated grassland if the skin temperature is 40°C? (b) What is the value of latent heat flux?
Use $\rho c_p \approx 1231(\text{W m}^{-2})/(\text{K m s}^{-1})$ for dry air at sea level.

SOLUTION

V (m/s) =	5	(a)	F_{H_S} (K m s^{-1}) =	**0.1**
T (°C) =	20	(b)	F_{E_S} (K m s^{-1}) =	**0.2**
T_s (°C) =	40	-or-		
unirrigated grass	0.001 for C_H	(a)	Q_H (W m^{-2}) =	123.1
Bowen ratio =	0.5	(b)	Q_E (W m^{-2}) =	246.2

Method: Use bulk aerodynamic method for (a),
then use Bowen ratio for (b).
For dynamic fluxes, at sea level, $\rho\ c_p$ (W m^{-2})/(K m s^{-1}) = 1231

a) Use eqn. (9.19a) with C$_H$ = 0.001 from Table 9.2 for unirrigated grass.

$$F_{Hs} = (0.001)\left(5 \text{ m s}^{-1}\right)(40 - 20°\text{C}) = 0.1 \text{ K·m s}^{-1}$$

b) Rearrange the Bowen ratio definition: $F_{Es} = F_{Hs}/B$, where $B = 0.5$ for grassland.

$$F_{Es} = (0.1 \text{ K·m s}^{-1})/0.5 = 0.2 \text{ K·m s}^{-1}$$

Alternately, these kinematic fluxes can be multiplied by $\rho{\cdot}c_p = 1232$ (W m^{-2})/(K·m s^{-1}): a) $Q_H = 123.1$ W m^{-2} and b) $Q_E = 246.2$ W m^{-2}.

9.24 As a cold, continental air mass passes over the Gulf Stream on a winter day, the temperature of the air in the ABL rises by 10 K over a distance of 300 km. Within this interval the ABL average depth is 1 km and the wind speed is 15 m s^{-1}. No condensation is taking place within the PBL and the radiative fluxes are negligible. Calculate the sensible heat flux from the sea surface.

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SOLUTION Use Eq. (9.31) and expand the total derivative:

$$\frac{d\theta}{dt} = \frac{\partial\theta}{\partial t} + U\frac{\partial\theta}{\partial x} = \frac{(1+A)\,F_{Hs}}{z_i}$$

At steady state, $\partial\theta/\partial t = 0$, leaving:

$$U\frac{\partial\theta}{\partial x} = \frac{(1+A)\,F_{Hs}}{z_i}$$

Solving for F_{Hs} gives:

$$F_{Hs}\frac{\left(15 \text{ m s}^{-1}\right)\cdot(10 \text{ K})\cdot(1000 \text{ m})}{(3\times10^5 \text{ m})\cdot(1.2)} = 0.417 \text{ K·m s}^{-1}$$

Multiply by $\rho{\cdot}c_p = 1231(\text{W m}^{-2})/(\text{K·m s}^{-1})$ to give the flux in dynamic units: $Q_H = 513$ W m^{-2}.

9.25 (a) If drag at the ground represents a loss of momentum from the mean wind, determine the sign of $\left(\overline{u'w'}\right)_s$ if the mean wind is from the west. Justify your result.

(b) Do the same for a wind from the east, remembering that drag still represents a momentum loss.

SOLUTION Assume wind = 0 a the ground, causing a wind shear just above the ground.

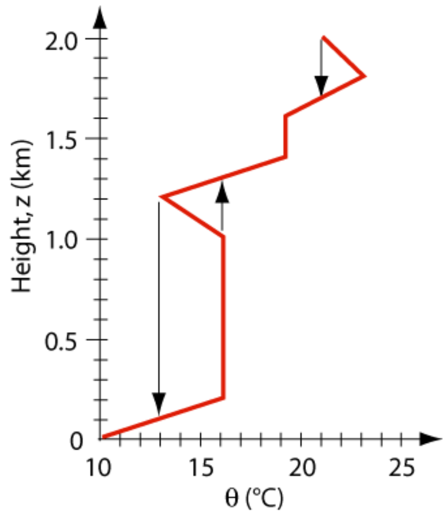
a) Justification: Wind from the west means positive U. If this positive momentum is transported downward (negative w'), then $\overline{u'w'} =$ negative based on the procedure of Fig. 9.8.
b) East wind is negative U. Downward transport (negative w') gives negative times negative. Thus, $\overline{u'w'}$ = positive.

9.26 Given the following temperature profile, determine and justify which layers of the atmosphere are statically stable, neutral, and unstable.

SOLUTION Plot θ versus z, then lift a hypothetical air parcel from each relative maximum in θ, and lower from each relative minimum, until hitting the sounding (see arrows in the figure) to identify unstable layers.

Outside of the unstable regions, identify as "stable" those regions for which $\partial\theta/\partial z > 0$, and identify as neutral regions where $\partial\theta/\partial z \sim 0$.

z (km)	θ (°C)
2	21
1.8	23
1.6	19
1.4	19
1.2	13
1	16
0.2	16
0	10



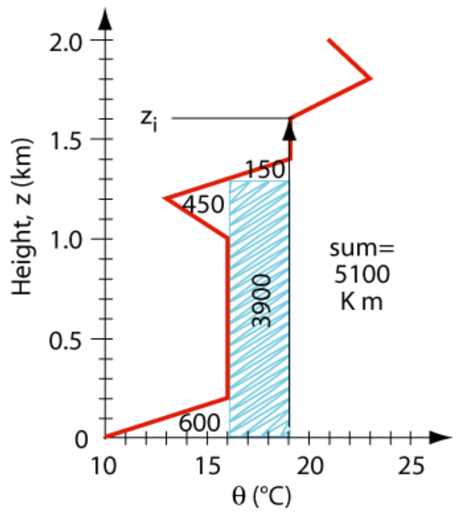
Unstable layers:
0.1 to 1.3 km
and
1.7 to 2.0 km

Stable layers:
0 to 0.1 km
and
1.3 to 1.4 km
and
1.6 to 1.7 km

Neutral layers
1.4 to 1.6 km

9.27 If the total accumulated heat flux from sunrise through sunset is 5100 K·m, then use the sounding in the previous exercise to estimate the depth and temperature of the mixed layer just before sunset.

SOLUTION Find the mixed-layer conditions such that the area between the original sounding and the new mixed-layer potential temperature equals the accumulated heat available = 5100 K·m. See the figure below, where the area was found by dividing the complex shape into simple rectangles and triangles with easy areas to calculate. Using trial and error, the mixed-layer potential temperature was gradually increased until the total area reached the desired value. The final values are $< \theta > =$ 19°C, which intercepts the sounding at 1.4 km. However, because of the neutral layer above 1.4 km, there is no resistance to thermals from the ground to rise up to z_i = 1.6 km.. Finally, recall that for temperature differences: $\Delta T(°\mathrm{C}) = \Delta T(\mathrm{K})$.



9.28 Given a smoke stack half the height of a valley, (a) Describe the path of the centerline of the smoke plume during day and night during fair weather. (b) Describe the centerline path of the smoke on a strongly windy day.

SOLUTION Assume the stack is in the center of a valley. (a) Daytime: initially a fanning plume with a subsiding centerline moves downstream due to return circulation from anabatic winds. Later, after the top of the mixed layer is higher than the stack, the plume will loop and quickly spread within the valley mixed layer, some of which would be drawn up the ridge slopes due to anabatic winds. At night, the plume would cone initially in the residual layer and remain above the smokestack, but later in the night it would fan as the centerline moves downstream in the mountain winds. (b) If the wind had a component along the valley axis, then the smoke would be channeled within the valley while rapidly coning due to intense turbulence. For across-valley winds, either the smoke would recirculate in a cavity, or would follow a mountain-wave trajectory over the downstream ridge.

9.29 If you know the temperature and humidity jump across the top of the mixed layer, and if you know only the surface heat flux (but not the surface moisture flux), show how you can calculate the entrained heat and moisture fluxes at the top of the mixed layer.

SOLUTION Use the Ball parameter, Eq. (9.30) in the text, and knowing the value of F_{Hs}, the entrained heat flux can be deterimined. Then, use Eq. (9.29) and eqn. (9.32) in the text to get $F_{Ezi} = -0.2 F_{Hs}\ \Delta q/\Delta\theta$.