Chapter 8

8.9 If $\omega = V/R_T$, the local angular velocity of the air moving along a cyclonically curving trajectory, and Ω is the angular velocity of the Earth's rotation, show that the flow is locally geostrophic if $\omega << \Omega$ and cyclostrophic if $\omega >> \Omega$.

The relative magnitudes of the centripetal acceleration and the Coriolis force are

$$\begin{aligned} \frac{V^2 R_T}{fV} &= \frac{\omega^2 R_T}{f\omega R_T} \\ &= \omega/f \end{aligned}$$

If $\omega/f >> 1$, the centripetal acceleration dominates and the flow is in cyclostrophic balance and if $\omega/f << 1$, the Coriolis force dominates and the flow is in geostrophic balance.

8.10 Show that the cyclonic shear of the flow in Fig. 8.31 contributes to the relative prominence of the warm frontal zone relative to the cold frontal zone. [**Hint**: rotate the right hand panel of Fig. 7.4 clockwise by 90° and apply it to the warm frontal zone.]

8.12 In Exercise 8.6 estimate the pressure deficit at the radius of maximum wind speed.

$$\begin{aligned} \delta p &= \frac{\rho v_0^2}{r_0^2} \int_0^{r_0} r dr \\ &= \frac{\rho v_0^2}{2} \end{aligned}$$

Substituting v_0 =100 m s^{-1}, $\rho = 1.25$ kg m^{-3}, then $\delta p = 1.25 \times 10^4$ Pa = 62.5 hPa.

8.13 Suppose that Dorothy's house had a cross sectional area of 200 m^2, a mean height of 5 m and a mass (including Dorothy and her dog) of 5 metric tons and that when the tornado passed overhead, the decrease in pressure with height was just enough to gently lift the house off the ground. Estimate the rate of decrease of pressure with height at ground level under the tornado. Assume an ambient air density of 1 kg m^{-3}.

If M is the mass of Dorothy's house and its contents, then

$$-\frac{1}{\rho}\frac{\partial p}{\partial z} = \frac{Mg}{A}$$

Substituting $\rho = 1$, $g \simeq 10$ m s^{-2}, $M = 5 \times 10^3$, and $A = 200$ m^2 yields

$$-\frac{\partial p}{\partial z} = 25 \text{ Pa m}^{-1} = 250 \text{ hPa km}^{-1}$$

8.14 The vertical velocity in a downdraft 500 m above the Earth's surface is 4 m s^{-1} and the radius of the downdraft is 3 km. Estimate the speed of the outflow from the base of the downdraft, averaged over the lowest kilometer. Neglect the vertical variation of density with height and use a value of 1 kg m^{-3}.

From the continuity equation, ignoring the variation of density with height in this relatively thin layer

$$\frac{\partial V}{\partial r} \sim \frac{\partial w}{\partial z}$$

where r is the radius of the downdraft. Substituting values

$$\frac{\partial V}{\partial r} \sim \frac{4 \text{ m s}^{-1}}{1 \text{ km}}$$

If V increases at this rate over a distance of 3 km, then $V = 12$ m s^{-1}.

8.15 If x is the direction of movement of the gust front and u is the velocity component in that direction, making use of (1.3) and (7.11) show that if the Coriolis force and friction are neglected, $\partial/\partial y = 0$, and the vertical velocity is zero,

$$\frac{d}{dx}\left(\frac{u^2}{2} + \frac{p}{\rho}\right) = 0$$

If U_e is the speed of the u component of the wind in the undisturbed environmental air in advance of the gust front, U_f is the speed of the u component at the gust front (i.e., at the peak of the pressure surge), and δp is the amplitude of the pressure surge, show that

$$\delta p = \rho_0 \left(U_f - U_e\right)^2$$

If δp =1 hPa, $\rho = 1.25$ kg m^{-3}, and $U_e = 5$ m s^{-1}, estimate the wind speed at the gust front, ignoring the effect of friction.

From the horizontal equation (7.11) with the Coriolis force and friction neglected

$$\frac{du}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial x}$$

Expanding the total derivative as in (1.3) and setting $\partial/\partial t = 0$ yields

$$u\frac{\partial u}{\partial x} = -\frac{1}{\rho}\frac{\partial p}{\partial x}$$

or, if $\partial \rho/\partial x << \partial p/\partial x$,

$$\frac{\partial}{\partial x}\left(u^2 - \frac{p}{\rho_0}\right) = 0$$

Integrating from the gust front forward in x into the indisturbed environmental air, we obtain

$$\delta p = \rho_0 \left(U_f - U_e\right)^2$$

Subsitituting values, we obtain

8.16 As boundary layer air spirals inward into the eye of a tropical cyclone, its temperature remains fixed at 27°C while the water vapor mixing ratio increases from 15 to 21 g kg^{-1}and the pressure drops from 1012 to 940 hPa. Estimate the resulting increase in equivalent potential temperature θ_e.

8.17 The lowest sea-level pressure ever observed in a tropical cyclone was 870 hPa in the center of Typhoon Tip in the western Pacific in 1979. Suppose that the 200-hPa pressure surface in this storm was flat. Estimate the ratio of the vertically (with respect to $\ln p$) averaged virtual temperature of the air in the eye of the storm to that in the large-scale environment, and the corresponding temperature difference.

Let T_i be the vertically-averaged virtual temperature inside the storm and T_o the vertically-averaged virtual temperature outside. Assume that the sea-level pressure outside the cyclone is close to the globally-averaged sea-level pressure of 1013 hPa. Based on the hypsometric equation (3.29), we can write

$$(R_d/g_0) \times T_i \times \ln\left(\frac{870}{200}\right) = (R_d/g_0) \times T_o \times \ln\left(\frac{1013}{200}\right)$$

From which

$$\frac{T_i}{T_o} = \frac{\ln(1013/200)}{\ln(870/200)} = \frac{1.622}{1.470} = 1.103 \qquad (1)$$

To estimate $T_i - T_o$ we need to assume a reference vertically-averaged (sea level to 200-hPa) virtual temperature. From the U.S. Standard Atmosphere plotted on the skew-T ln p chart, the 200-hPa level corresponds to ∼12 km. Hence, throm the hypsometric equation (3.29)

$$12 \times 10^3 = (R_d/g_0) T_o \ln\frac{1013}{200}$$

Substituting $R_d = 287$ and $g_0 = 9.8$ and solving, we obtain $T_o = 252$ K. From (1)

$$\begin{aligned} T_i - T_o &= (1.103 - 1.000)T_o \\ &= 0.103 T_o \\ &= 26 \text{ K} \end{aligned}$$

8.18 Just outside the eyewall of an intense tropical cyclone, at a radius of 10 km, the azimuthal wind speed is 60 m s^{-1}. Estimate the radial pressure gradient. Assume an air density of 1.1 kg m^{-3}.

Assume that the azimuthal wind component u is in cyclostrophic balance.

$$\frac{\partial p}{\partial r} = \rho \frac{u^2}{r}$$

Substituting values we obtain

$$\begin{aligned}\frac{\partial p}{\partial r} &= 1.1 \times \frac{60^2}{10 \text{ km}} \\ &= 396 \text{ Pa km}^{-1}\end{aligned}$$

8.19 If the azimuthal wind speed in the previous exercise decreases with height from 50 m s^{-1} at the 500-hPa level to zero at the 250-hPa level, estimate the mean radial gradient of virtual temperature within this layer.

On the 500-hPa surface

$$\begin{aligned}\frac{\partial \Phi}{\partial r} &= g\frac{\partial Z}{\partial r} = \frac{50^2}{10 \text{ km}} \\ \frac{\partial Z}{\partial r} &= 25.5 \text{ m km}^{-1}\end{aligned}$$

On the 250 hPa surface $\partial Z/\partial r = 0$. Hence, for the 500-250-hPa layer, $\partial/\partial r\,(\delta Z) = 25.5$ m km^{-1} or .0255. From the hypsometric equation (3.29)

$$\frac{\partial}{\partial r}\frac{R_d}{g_0}T_v \ln\frac{500}{250} = 25.5 \text{ m km}^{-1}$$

Substituting $R_d/g_0 = 29.3$ and $\ln(500/250) = 0.693$ yields

$$\begin{aligned}\frac{\partial T_v}{\partial r} &= \frac{25.5}{29.3 \times 0.693} \\ &= 1.256^{\circ}\text{C km}^{-1}\end{aligned}$$

8.20 Consider an axially symmetric tropical cyclone that forms at a latitude of 15° in a large-scale environment in which the air is initially at rest. From how far out would the low-level inflow have to come in order to develop an azimuthal wind speed of 40 m s^{-1} at a radius of 20 km in the absence of frictional drag?

In the absence of frictional drag, angular momentum is conserved in an inertial coordinate system. For a circular ring of air concentric with the cyclone the angular momentum due to the Earth's rotation is equal to the Ω times the square of the radius times the sine of the latitude angle and the angular momentum associated with the relative motion of the air is equal to the tangential wind speed u times the radius. Hence

$$r_1\,(u_1 + \Omega r_1 \sin\phi) = r_2\,(u_2 + \Omega r_2 \sin\phi)$$

and if radius R is sufficiently large that the relative wiund speed is negligible

$$\Omega R^2 \sin\phi = r\,(u + \Omega r \sin\phi)$$

Substituting values $\Omega \sin(15°) = 1.87 \times 10^{-5}$ s^{-1}, $r = 20 \times 10^3$ m, and $u = 40$ m s^{-1} yields

$$\begin{aligned} R^2 &= \frac{\left(20 \ \times 10^3 \text{ m}\right) \ \left[40 \text{ m s}^{-1} + 1.87 \times 10^{-5} \text{ s}^{-1} \left(20 \times 10^3 \text{ m}\right)\right]}{1.87 \times 10^{-5} \text{ s}^{-1}} \\ &= \frac{800 + 7.48) \times 10^3 \text{ m}^2 \text{ s}^{-1}}{1.87 \times 10^{-5} \text{ s}^{-1}} \\ &= 4.32 \times 10^{10} \text{ m}^2 \end{aligned}$$

Hence, $R = 208$ km.

8.21 In the eye of an intense tropical cyclone the sea-level pressure is 60 hPa lower than in the large-scale environment. Estimate the elevation of sea level due to the hydrostatic adjustment to the low pressure.

A pressure perturbation δp relative to mean sea level is accompanied by a hydrostatic sea level displacement δz; i.e.,

$$\delta p = -\rho_w g \delta z$$

where ρ_w is the density of sea water. Hence,

$$\delta z = \frac{-\delta p}{\rho_w g}$$

Substituting values yields

$$\begin{aligned} \delta z &= \frac{-6000 \text{ Pa}}{10^3 \times 9.8} \\ &= 61 \text{ cm} \end{aligned}$$