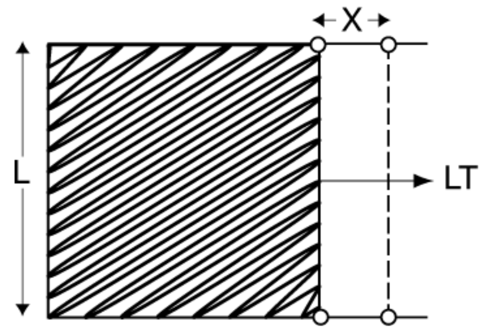
Chapter 6

6.9



Let

$$\begin{aligned} T &= \text{surface tension} \\ &= \text{force per unit length on wire} \end{aligned}$$

Then, work done in moving wire of length L through distance $x = (TL)\,x$

$$\begin{aligned} \frac{\text{Work done}}{\text{Increase in area of soap film}} &= \frac{(TL)\,x}{Lx} = T \\ \therefore \text{ Surface energy} &= \text{Surface tension} \\ (\text{Units: J m}^{-2} &= \frac{\text{Force}}{\text{length}} = \frac{m\ell t^{-2}}{\ell} = mt^{-2} \\ \frac{m\ell t^{-2}\ell}{\ell^2} &= mt^{-2} \\ mt^{-2} &= mt^{-2}) \end{aligned}$$

6.10 From eqn. (6.5)

$$\begin{aligned} \ln\frac{e}{e_s} &= \frac{2\sigma}{nkTr} \\ \sigma &= 0.76 \text{ J m}^{-2} \\ n &= 3.3 \times 10^{28} \text{ m}^{-3} \\ k &= 1.38 \times 10^{-23} \\ T &= 273^\circ\text{K} \\ r &= 0.2 \times 10^{-6} \text{ m} \\ \therefore \ln\frac{e}{e_s} &= \frac{2\,(0.76)}{(3.3 \times 10^{28})\,(1.38 \times 10^{-23})\,(273)\,(0.2 \times 10^{-6})} \\ &= 6.113 \times 10^{-3} \\ \therefore \frac{e}{e_s} &= 1.00613 \\ \therefore \underline{\text{Relative humidity}} &= \underline{100.6\%} \end{aligned}$$

6.11 See Fig. 6.3.

6.12 From eqn. (6.8)

$$\frac{e'}{e_s} = \left[\exp \frac{2\sigma'}{n'kTr}\right] \left[1 + \frac{imM_w}{M_s\left(\frac{4}{3}\pi r^3 \rho' - m\right)}\right]^{-1}$$

For a very weak solution $m \ll \frac{4}{3}\pi r^3 \rho'$. Also, since $\frac{2\sigma'}{n'kTr} \ll 1$, we have

$$\begin{aligned} \frac{e'}{e_s} &\simeq \left[1 + \frac{2\sigma'}{n'kTr}\right] \left[1 - \frac{imM_w}{\frac{4}{3}\pi M_s r^3 \rho}\right] \\ &\simeq 1 + \frac{2\sigma'}{n'kTr} - \frac{imM_w}{\frac{4}{3}M_s \pi r^3 \rho'} - \underbrace{[\ ]\frac{1}{r_4}}_{\text{very small}} \end{aligned}$$

$$\boxed{\frac{e'}{e_s} \simeq 1 + \frac{a}{r} - b}$$

where,

$$a = \frac{2\sigma'}{n'kT} \text{ and } b = \frac{imM_w}{\frac{4}{3}\pi M_s \rho'}$$

Second term represents the effect of the curvature of the drop in increasing e'. Third term represents the effect of dissolved salt in decreasing e'/e_s. Peak in the Köhler curve (i.e., in e'/e_s) with r occurs when

$$\frac{d}{dr}\left(\frac{e'}{e_s}\right) = 0,$$

that is, when

$$0 \simeq -a - \frac{3b}{r^2}$$

or, when

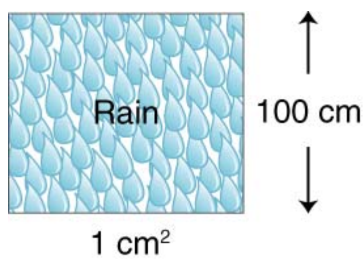
$$\boxed{r^2 \simeq \frac{3b}{a}}$$

The magnitude of $\frac{e'}{e_s}$ at its peak value is

$$\begin{aligned}
\left(\frac{e'}{e_s}\right)_{\max} &\simeq 1 + a\left(\frac{a}{3b}\right)^{1/2} - b\left(\frac{a}{3b}\right)^{3/2} \\
&\simeq 1 + \left(\frac{a^3}{3b}\right)^{1/2} - \left(\frac{b^{2/3}a}{3b}\right)^{3/2} \\
&\simeq 1 + \left(\frac{a^3}{3b}\right)^{1/2} - \left(\frac{a^3}{3^3 b}\right)^{1/2} \\
&\simeq 1 + \left(\frac{a^3}{3b}\right)^{1/2} - \frac{1}{3}\left(\frac{a^3}{3b}\right)^{1/2} \\
&\simeq 1 + \frac{2}{3}\left(\frac{a^3}{3b}\right)^{1/2}
\end{aligned}$$

$$\boxed{\left(\frac{e'}{e_s}\right)_{\max} \simeq 1 + \left(\frac{4a^3}{27b}\right)^{1/2}}$$

6.13



Volume (or mass in grams) of rainwater collected over 1 cm^2 in 1 year = 100 cm^3 (or grams). If N = number of cloud droplets/cm^3

$$\begin{aligned}
N\,(\text{mass of a single cloud droplet}) &= LWC\left(\text{in g cm}^{-3}\right) \\
&= 0.3 \times 10^{-6} \\
\therefore\ \text{mass of a single cloud droplet (in grams)} &= \frac{0.3 \times 10^{-6}}{N}
\end{aligned}$$

But, in 1 year 100 grams of rain is collected

$\therefore$ Number of cloud droplets removed by rain per year

$$\begin{aligned}
&= \frac{100}{\text{mass of single drop}} = \frac{100}{0.3 \times 10^{-6}/N} \\
&= \frac{100\ N}{0.3 \times 10^{-6}} \\
&= 0.3 \times 10^{9}\ N \\
&\simeq 3 \times 10^{8}\ N
\end{aligned}$$

Number of cloud droplets removed per second

$$\simeq \frac{3 \times 10^8 \ N}{3.65 \times 24 \times 60 \times 60}$$
$$\simeq 9.5 \ N$$

Number of CCN in column of atmosphere with 1 cm^2 areal cross sectional and 5 km high

$$= \left(1 \times 5 \times 10^5\right) N$$

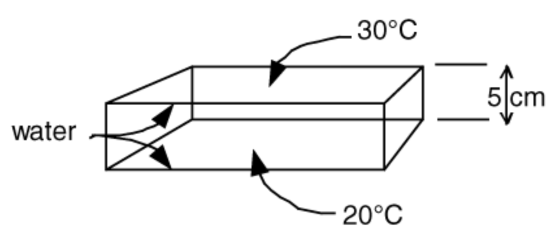
$$\therefore \text{Fraction of CCN removed/sec} = \frac{9.5 \ N}{5 \times 10^5 \ N} = 1.9 \times 10^{-5}/\text{sec}$$

$$\text{Fraction of CCN removed/day} = \left(1.9 \times 10^{-5}\right)(24 \times 60 \times 60) = 1.64$$

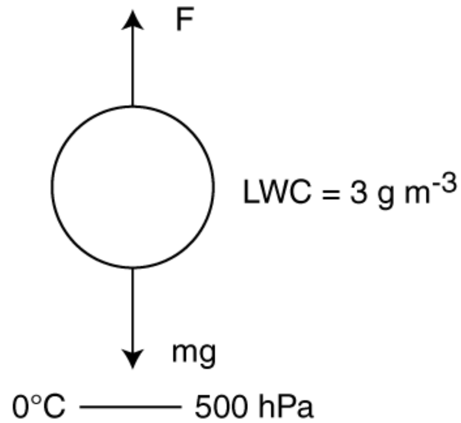
Hence, all of CCN are removed in less than 1 day. (*Note*: From (____)

$$\text{Residence time} \equiv \frac{\text{Amount of species}}{\text{Rate of removal of species}} = \frac{5 \times 10^5 \ N}{9.5 \ N} \text{ sec} = 0.52 \times 10^5 \text{ sec} = 14.5 \text{ hrs} = 15 \text{ hrs}$$

6.14



6.15



When drops are falling at terminal fall speed, the frictional drag per unit mass of air is

(a) F_d = Downward force on drops/unit mass of air

$$= \left(\frac{\text{mass of drops}}{\text{mass of air}}\right) \text{ g}$$

Since, air density $= \rho = \dfrac{\text{mass of air}}{\text{volume of air}}$

$$\begin{aligned} F_d &= \frac{(\text{mass of drops})}{(\text{volume of air})} \frac{g}{\rho} \\ &= \left[3 \times 10^{-3} \left(\text{kg m}^{-3}\right)\right] \frac{g}{\rho} \end{aligned}$$

Also, for air

$$\begin{aligned} p &= R_d \rho T \\ \therefore F_d &= \left(3 \times 10^{-3}\right) g \frac{R_d T}{p} \\ &= \frac{\left(3 \times 10^{-3}\right)(9.81)(287)(273)}{500 \times 10^2} \\ &= \frac{3 \times 9.81 \times 287 \times 273}{5} \times 10^{-7} \\ &= 461174 \times 10^{-7} \end{aligned}$$

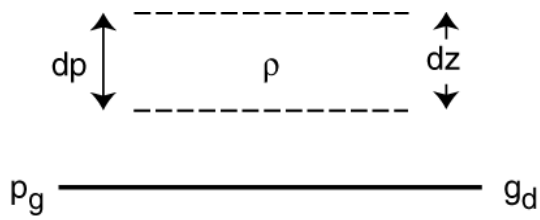
$$\boxed{\boldsymbol{F_d = 0.0461\ N\ \text{kg}^{-1}}}$$

(b) Without drops being present

Downward force acting on a unit mass of air $= g$

When drops are present

$$\begin{aligned}\text{Downward force acting on a unit mass of air} &= (g+0.0461)\\ &= (9.81+0.0461)\\ &= 9.8561\ N\ \text{kg}^{-1}\end{aligned}$$



$$\begin{aligned}\text{Pressure at ground} &= p_g = -\int_{p_g}^{o} dp\\ \text{But } \frac{dp}{dz} &= -\rho g\\ \therefore p_g &= \int_o^{\infty} \rho g dz = g\int_o^{\infty} \rho dz \qquad (1)\end{aligned}$$

Now the pressure of $3g/\text{m}^3$ $\left(= 3\times 10^{-3}\ \text{kg/m}^3\right)$ of water in air, will have negligible effect on the density of air, which is ~ 1.275 kg m^{-3} at surface. Therefore from (1) we can write

$$p_g \propto g$$

Similarly, at cloud height

$$p \propto g$$

Hence,

$$\begin{aligned}\frac{p\,(\text{air alone}))}{p\,(\text{air + cloud water})} &= \frac{g}{g+0.0461}\\ &= \frac{9.81}{9.8561} \qquad (2)\end{aligned}$$

Since

$$p = R_d \rho T$$

the air in question (i.e., air containing water vapor) has density

$$\rho = \frac{p\,(\text{air + cloud water})}{R_d T} \qquad (3)$$

If the air had no cloud water, but is required to have density ρ given by (3), then

$$\text{Density} = \frac{p\,(\text{air only})}{R_d T_v}$$

or,

$$\frac{p\,(\text{air} + \text{cloud water})}{R_d T} = \frac{p\,(\text{air alone})}{R_d T_v}$$

where T_v is the virtual temperature require g the air alone is to have density ρ.

$$\therefore T_v = \frac{p\,(\text{air alone})}{p\,(\text{air} + \text{cloud water})} T \tag{4}$$

From (2) and (4)

$$\begin{aligned} T_v &= \frac{9.81}{9.8561} T \\ &= 0.9953\,(273) \\ &= 271.72 \text{ K} \end{aligned}$$

Therefore, the (*negative*) virtual temeprature correction is $273 - 271.72 = \underline{\underline{1.28°\text{C}}}$.

6.16 (a) The cloud liquid water content (LWC) is given by

$$LWC = \frac{4}{3}\pi\rho_\ell \int_o^\infty r^3 n(r) dr \tag{1}$$

where $n(r)$ is the concentration of droplets or radius r, and

$$N = \int_o^\infty n(r) dr \tag{2}$$

From (1) and (2)

$$LWC = \frac{4}{3}\pi\rho_\ell \left(\overline{r}\right)^3 N$$

where $\overline{r}$ is the mean droplet radius $\simeq r_e$. Therefore

$$\boxed{LWC = \frac{4}{3}\pi\rho_\ell r_e^3 N} \tag{3}$$

(b) Also,

$$\tau_c = 2\pi h r_e^2 N \tag{4}$$

From (3) and (4)

$$LWC = \frac{4}{3}\pi\rho_\ell r_e^3 \frac{\tau_c}{2\pi h r_e^2}$$

or,

$$\boxed{LWC = \frac{2}{3}\rho_\ell \frac{r_e \tau_c}{h}} \tag{5}$$

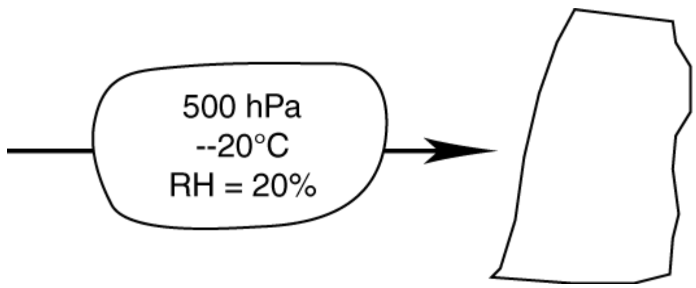
(c)

$$LWP = LWC\left(\text{kg m}^{-3}\right) h\,(m) \qquad (6)$$

From (5) and (6)

$$\boxed{LWP = \frac{2}{3}\rho_\ell r_e \tau_c}$$

6.17



Entrained air will be coooled to its *wet-bulb temperature*. From skew $T - \ln p$ chart: At 500 hPa and $T = 253$, saturation mixing ratio = 1.6 g/kg. Therefore,

$$\begin{aligned} \text{Relative humidity} &= \frac{\text{actual mixing ratio}}{\text{saturation mixing ratio}} 100 \\ \therefore 20 &= \frac{\text{actual mixing ratio}}{1.6} 100 \end{aligned}$$

$$\begin{aligned} \therefore \text{Actual mixing ratio} &= 1.6 \times 0.2 \\ &= 0.32 \text{ g/kg} \end{aligned}$$

$\therefore$ Dew point of air before it is entrained into cloud $= -37°\text{C}$

Using Norman's rule, the wet-bulb temperature of the entrained air $= -23°\text{C}$

If air parcel is brought down along a saturated adiabat to 1000 hPa, its temperature is found to be 12°C.

If final relative humidity is 50%, by trial and error from chart (by going down a saturated adiabat and then a dry adiabat) final temperature is 19°C.

6.18 (a) From eqn. (6.18) in Chapter 6:

$$\frac{d\theta'}{\theta'} = -\frac{L_v}{c_p T'} dw_s - \left[\frac{T - T'}{T'} + \frac{L_v}{c_p T'}\left(w_s - w\right)\right] \frac{dm}{m}$$

For no condensation ($dw_s = 0$) and no entrainment ($dm = 0$), and (5.28) becomes $\dfrac{d\theta'}{\theta} = 0$ or $d\theta' = 0$. This is the dry air ascending adiabatically case.

(b) For condensation but no entrainment ($dm = 0$)

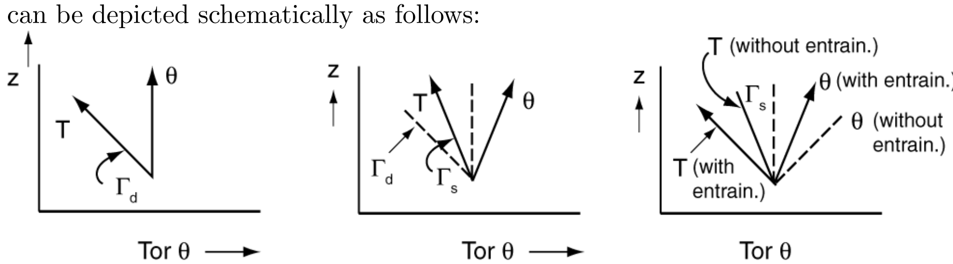
$$\frac{d\theta'}{\theta'} = -\frac{L_v dw_s}{c_p T'}$$

or

$$c_p \frac{d\theta'}{\theta'} = \frac{dQ}{T'} = ds$$

which is equivalent to (3.98).

(c) For condensation and entrainment, the terms inside [ ] in (7.18) are both positive. Since, $-\frac{L_v dw_s}{c_p T'}$ is positive, with entrainment $\frac{d\theta'}{\theta'}$ is less than without entrainment. That is, the rate of decrease of θ' with increasing height is less with entrainment. The three situations can be depicted schematically as follows:



(i) Dry air ascending (ii) Saturated air ascending (no entrainment) (iii) Saturated air ascending with entrainment

See from (iii) that entrainment causes T to decrease faster with z than for Γ_s.

6.19 Let the radius of the termal at height z above the ground be r, then

$$r = \alpha z \quad (1)$$

$\alpha =$ constant. The entrainment rate is

$$\frac{1}{m}\frac{dm}{dt} = \frac{1}{4\pi r^3 \rho}\frac{d}{dt}\left(\frac{4}{3}\pi r^3 \rho\right) = \frac{3}{r}\frac{dr}{dt} \quad (2)$$

From (1) and (2)

$$\boxed{\frac{1}{m}\frac{dm}{dt} = \frac{3\alpha}{r}\frac{dz}{dt}}$$

6.20 (a) For ascent with no condensation LWC = 0, therefore:

$$\frac{dS}{dt} = Q_1 \frac{dz}{dt} \quad (1)$$

since,

$$S = \frac{e}{e_s}$$

$$\frac{dS}{dt} = \left(e_s \frac{de}{dt} - e \frac{de_s}{dt} \right) \Big/ e_s^2 \tag{2}$$

We will first evaluate $\dfrac{de}{dt}$

$$e = \frac{w}{\varepsilon + w} p$$

where, w = mixing ratio, which is constant if there is no condensation.

$$\therefore \frac{de}{dt} = \frac{w}{\varepsilon + w} \frac{dp}{dt}$$

or

$$\frac{de}{dt} = \frac{w}{\varepsilon + w} \frac{dp}{dz} \frac{dz}{dt} \tag{3}$$

But,

$$\frac{dp}{dz} = -g\rho \tag{4}$$

and

$$p = R_d \rho T \qquad (T = T_v) \tag{5}$$

From (3), (4), (5):

$$\frac{de}{dt} = -\frac{eg}{R_d T} \frac{dz}{dt} \tag{6}$$

We will now evaluate $\dfrac{de_s}{dt}$

From the Clausius-Clapeyron eqn:

$$\frac{de_s}{dT} = \frac{L_v}{T(\alpha_2 - \alpha_1)} \simeq \frac{L_v}{T(\alpha_2)}$$

and,

$$e_s = R_v \frac{1}{\alpha_2} T$$

$$\therefore \frac{de_s}{dT} = \frac{L_v e_s}{R_v T^2}$$

$$\therefore \frac{de_s}{dt} = \frac{L_v e_s}{R_v T^2} \frac{dT}{dt}$$

or,

$$\frac{de_s}{dt} = \frac{L_v e_s}{R_v T^2} \frac{dT}{dz} \frac{dz}{dt}$$

But,

$$-\frac{dT}{dz} = +\frac{g}{c_p} = \begin{array}{l}\text{Dry adiabatic lapse rate} \\ \text{(because there is no condensation)}\end{array}$$

$$\therefore \frac{de_s}{dt} = -\frac{L_v e_s}{R_v T^2} \frac{g}{c_p} \frac{dz}{dt} \tag{7}$$

From (2), (6) and (7):

$$\frac{dS}{dt} = \frac{1}{T}\frac{e}{e_s}\left(\frac{\varepsilon L_v g}{R_d c_p T} - \frac{g}{R_d}\right)\frac{dz}{dt}$$

Since $\frac{e}{e_s} \simeq 1$

$$\frac{dS}{dt} \simeq \frac{1}{T}\left(\frac{\varepsilon L_v g}{R_d c_p T} - \frac{g}{R_d}\right)\frac{dz}{dt} \tag{8}$$

Comparing (1) and (8):

$$\boxed{Q_1 = \frac{g}{TR_d}\left(\frac{\varepsilon L_v}{c_p T} - 1\right)}$$

(b) If we assume no vertical air motion $\left(\frac{dz}{dt} \simeq 0 \text{ and } p = \text{constant}\right)$ while condensation occurs, then:

$$\frac{dS}{dt} = -Q_2 \frac{d\,(LWC)}{dt} \tag{9}$$

Equation (2) still holds, and we will now evaluate $\frac{de}{dt}$ for this case.

$$e = \frac{w}{\varepsilon + w}p$$

w now varies, but p is constant. Therefore,

$$\frac{de}{dt} = \frac{\varepsilon}{(\varepsilon + w)^2}\frac{dw}{dt}p$$

But,

$$\begin{aligned} \frac{dw}{dt} &= -\frac{d\,(LWC)}{dt} \\ \therefore \frac{de}{dt} &= -\frac{\varepsilon}{(\varepsilon + w)^2}p\frac{d\,(LWC)}{dt} \end{aligned}$$

Since

$$p = R_d \rho T \qquad (T = T_v)$$

and

$$\varepsilon \gg w$$

$$\frac{de}{dt} \simeq -\frac{1}{\varepsilon}\rho R_d T \frac{d\,(LWC)}{dt} \tag{10}$$

We now evaluate $\frac{de_s}{dt}$ for this case. As before, from the Clausius-Clapeyron eqn. and the gas eqn. for vater vapor, we get:

$$\begin{aligned}\frac{de_s}{dt} &\simeq \frac{L_v e_s}{R_v T^2}\frac{dT}{dt} \\ &= \frac{L_v e_s}{R_v T^2}\frac{dT}{d\,(LWC)}\frac{d\,(LWC)}{dt}\end{aligned} \qquad (11)$$

But,

$$d\,(LWC) = -dw$$

and

$$dQ = L_v d\,(LWC) = -L_v dw$$

also,

$$dQ = c_p dT$$

Hence,

$$c_p dT = -L_v dw$$

or

$$dT = -\frac{L_v dw}{c_p} \qquad (12)$$

From (11) and (12):

$$\frac{de_s}{dt} = +\frac{L_v^2 e_s}{R_v T^2 c_p}\frac{d\,(LWC)}{dt} \qquad (13)$$

From (2), (10) and (13):

$$\frac{dS}{dt} = \left(e_s\frac{de}{dt} - e\frac{de_s}{dt}\right)\Big/\, e_s^2$$

Therefore,

$$\frac{dS}{dt} = -\frac{1}{\varepsilon e_s}\rho R_d T\frac{d\,(LWC)}{dt} - \frac{e}{e_s}\frac{L_v^2}{R_v T^2 c_p}\frac{d\,(LWC)}{dt}$$

Substitute

$$p = \rho R_d T \text{ so that } T = p/\,R_d\rho$$

$$\therefore \frac{dS}{dt} = -\frac{1}{\varepsilon e_s}\rho R_d T\frac{d\,(LWC)}{dt} - \frac{e}{e_s}\frac{L_v^2 R_d\rho}{R_v T c_p p}\frac{d\,(LWC)}{dt}$$

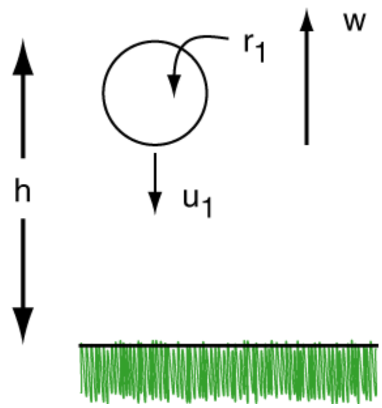
Or, since $\frac{e}{e_s} \simeq 1$,

$$\frac{dS}{dt} \simeq -\rho\left[\frac{R_d T}{\varepsilon e_s} + \frac{\varepsilon L_v^2}{T p c_p}\right]\frac{d\,(LWC)}{dt} \qquad (14)$$

From (9) and (14):

$$\boxed{Q_2 = \rho\left[\frac{R_d T}{\varepsilon e_s} + \frac{\varepsilon L_v^2}{p T c_p}\right]}$$

6.21



$$\frac{dh}{dt} = w - u_1 \quad (1)$$

$$u_1 = \frac{2g\rho_L r_1^2}{9\eta} \quad (2)$$

From (6.21)

$$r_1 \frac{dr_1}{dt} = G_\ell S$$

$$\therefore r_1^2 = 2G_\ell S t \quad (3)$$

$\therefore$ From (1), (2) and (3)

$$\frac{dh}{dt} = w - \frac{4g\rho_L G_\ell S}{9\eta} t$$

$$\therefore \int_o^h dh = \int_o^t dt - \frac{4g\rho_L S G_\ell}{9\eta} \int_o^t t dt$$

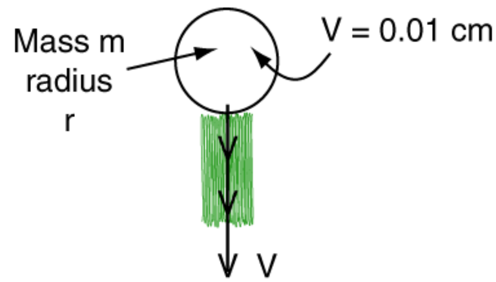
$$h = wt - \frac{4g\rho_L S G_\ell}{9\eta} \frac{t^2}{2}$$

$$\boxed{p = wt - \frac{2g\rho_L S G_\ell t^2}{9\eta}}$$

6.22 Use the skew $T - \ln p$ chart as described in Exercise 3.10.

Entrainment reduces LWC below adiabatic value. Accumulation of LWC (e.g., when fall speed of drops = updraft of air) could cause LWC to increase above adiabatic values in some regions of the cloud.

6.23



$$\frac{dm}{dt} = \pi r^2 V E_e \left(LWC\right)$$

But,

$$m = \frac{4}{3}\pi r^3 \rho_2$$

$$\therefore \rho_L \frac{4}{3}\pi^3 r^2 \frac{dr}{dt} = \pi r^2 V E_c \left(LWC\right)$$

$$\therefore \frac{dr}{dt} = \frac{VE_c}{4\rho_L}\left(LWC\right)$$

$$LWC = \frac{4}{3}\pi\left(0.001\right)^3 \rho_L 100 \left(\text{in g cm}^{-3}\right)$$

$$\text{where } \rho_L = \text{density of liquid water in g cm}^{-3} = 1$$

$$\therefore LWC = \frac{4}{3}\pi 10^{-7} \text{ g cm}^{-3}$$

Also, $E_c = 1$ and $V = \left(6 \times 10^3\right) r$ where V is in cm s^{-1} and r in cm.

$$\therefore \frac{dr}{dt} = \frac{6 \times 10^3 r \frac{4}{3}\pi 10^{-7} \ 0.8}{4}$$

or

$$\therefore \frac{dr}{r} = 1.6\pi 10^{-4} t$$

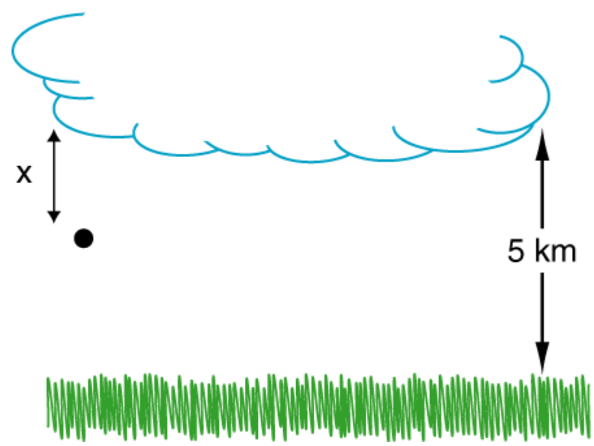
$$\int_{0.01 \text{ cm}}^{0.1 \text{ cm}} \frac{dr}{r} = 1.6\pi 10^{-4} \int_o^t dt = 1.6\pi 10^{-4} t$$

$$\therefore \frac{\ln 0.1 - \ln 0.01}{1.6\pi d 10^{-4}} = t$$

$$t = 4579 \text{ secs}$$

$$\underline{t = 76.3 \text{ mins}}$$

6.24



$$V = 6 \times 10^3 r$$

where V is in cm s^{-1} and r in cm

$$\therefore V = \left(6 \times 10^3\right) 10^{-4} r = 0.6r$$

where V is in cm s^{-1} and r in μm. Let x = distance of drop (in cm) below cloud base, then

$$\frac{dx}{dt} = V = 0.6r \qquad \text{(A)}$$

where r is in μm.

$$\therefore dx = 0.6r\ dt \ \ (x \text{ in cm } r \text{ in } \mu\text{m})$$

From (6.21)

$$r\frac{dr}{dt} = G_\ell S = \left(7 \times 10^2\right) \qquad \underset{\text{as a fraction}}{\overset{S}{\uparrow}} \qquad (r \text{ in } \mu\text{m}) \qquad \text{(B)}$$

$$\therefore rdr = \left(7 \times 10^2 S\right) \frac{dx\,(\text{cm})}{0.6r}$$

or

$$r^2 dr = \frac{\left(7 \times 10^2 S\right)}{0.6} dx$$

$$\therefore \int_{1000\ \mu\text{m}}^{R(\mu\text{m})} r^2 dr = \frac{700S}{0.6} \int_{0\ \text{cm}}^{5\times10^5\ \text{cm}} dx$$

$$\left[\frac{\Gamma^3}{3}\right]_{1000}^{R(\mu\text{m})} = \frac{700S}{0.6}\left(5 \times 10^5\right)$$

If $RH = 60\%$ the supersaturation is (as a fraction)

$$\frac{e - e_s}{e_s} = \frac{e}{e_s} - 1 = \underset{\text{(as a fraction)}}{RH} - 1 = 0.6 - 1 = -0.4$$

$$\therefore S = -0.4$$

$$\begin{aligned}
\therefore \left[\frac{r^3}{3}\right]_{1000}^{R(\mu\text{m})} &= \frac{700\,(-0.4)}{0.6}\left(5 \times 10^5\right) \\
\frac{R^3}{3} - \frac{1000^3}{3} &= \frac{-7\,(0.4)\,(5)\,10^7}{0.6} \\
&= -23.33 \times 10^7 \\
R^3 &= 10^9 - 3\left(23.33 \times 10^7\right) \\
&= 1 -^{9} -0.7 \times 10^9 \\
&= 0.3 \times 10^9 \\
\therefore R &= 3\sqrt{0.3} \times 10^3 \\
&= 0.6694 \times 10^3 \ \mu\text{m}
\end{aligned}$$

$$\boxed{\begin{array}{c}\text{Radius} \\ \text{at cloud} \\ \text{base}\end{array} = 0.67 \text{ mm}}$$

Time taken: And from (B) above

$$dt = \frac{r dr}{700 S_T}$$

$\therefore \int_{1000\ \mu\text{m}}^{r} r dr = 700S \int_{o}^{dt}$ where T = time (in secs) for drop to reach ground

$$\begin{aligned}
\left[\frac{r^2}{2}\right]_{1000}^{r} &= 700ST \\
\frac{r^2}{2} - \frac{1000^2}{2} &= 700ST \\
\therefore r^2 &= 1000^2 + 2\,(700)\,ST
\end{aligned}$$

But from (A) above

$$\frac{dx}{dt} = 0.6r$$

Hence

$$\begin{aligned}
\frac{dx}{dt} &= 0.6\left(10^6 + 1400St\right)^{1/2} \\
\therefore \int_o^{5\times10^5\ \text{cm}} dx &= 0.6\int_o^T \left(10^6 + 1400St\right)^{1/2} dt \\
5\times10^5 &= 0.6\left[\frac{2}{3}\left(10^6 + 1400St\right)^{3/2}\frac{1}{1400S}\right]_o^T \\
&= 0.6\left[\frac{2}{4200S}\left(10^6 + 1400ST\right)^{3/2} - \frac{2\times10^9}{4200S}\right]
\end{aligned}$$

But $S = -0.4$

$$\therefore 5\times10^5 = \frac{1.2}{4200\,(-0.4)}\left(10^6 = 560T\right)^{3/2} - \frac{1.2\times10^9}{4200\,(-0.4)}$$

$$\left[\frac{5\times10^5\times4200\,(-0.4)}{1.2} + 10^9\right]^{2/3} = 10^6 = 560T$$

$$\begin{aligned}
\therefore T &= \frac{10^6}{560} - \frac{\left(-7\times10^8 + 10^9\right)^{2/3}}{560} \\
&= \frac{10^6}{560} - \left(-0.7\times10^9 + 10^9\right)^{2/3}\Big/560 \\
&= 1785.7 - \left(0.3\times10^9\right)^{2/3}\Big/560 \\
&= 1785.714 - (0.3)^{2/3}\,10^6\big/560
\end{aligned}$$

$$\begin{aligned}
T &= 1785.714 - (0.6694)^2\,\frac{10^6}{560} \\
&= 1785.714 - 800.172 \\
&= 985.54 \text{ secs}
\end{aligned}$$

$$\boxed{T = 16.4 \text{ mins}}$$

6.25 (a)

$$\begin{aligned}
\text{Let } N &= \text{total number of drops} \\
N_t &= \text{number of drops frozen at time } t
\end{aligned}$$

Then

$$P\,(V,t) = \frac{N_t}{N}$$

Number of drops that nucleat between time o and $t+dt$ is

$$N_{t+dt} = N_t + (N - N_t)\,VJ_{LS}dt$$

Dividing both sides by N

$$P(V, t + dt) = P(V, t) + [1 - P(V, t) V J_{LS} dt]$$

Since

$$P(V, t + dt) = P(V, t) + \frac{d}{dt}[P(V, t)] dt$$

it follows that

$$\frac{d}{dt} P(V, t) = [1 - P(V, t)] V J_{LS}$$

Hence,

$$\int_o^{P(V,t)} \frac{dP(V, t)}{1 - P(V, t)} = \int_o^t V J_{LS} dt$$

$$\therefore \ln[1 - P(V, t)] = -\int_o^t V J_{LS} dt$$

But $\beta = \dfrac{dT}{dt}$

$$\therefore \boxed{\ln[1 - P(V, t)] = -\frac{V}{\beta} \int_o^{T_t} J_{LS} dT}$$

where T_t is temperature at time t.

(b) From the equation following (6.34) in Exercise (6.4):

$$-n = \ln(1 - P)$$

$$\therefore -\ln(1 - P) = \frac{4}{3}\pi \left(\frac{D}{2}\right)^3 10^3 \exp a(T_1 - T)$$

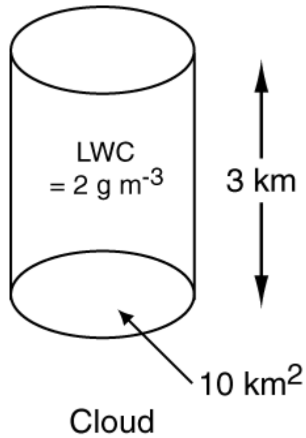
Since from (a) above

$$-\ln(1 - P) \quad \propto \quad \frac{V}{\beta}$$

$$\frac{V}{\beta} \quad \propto \quad \exp a(T_1 - T)$$

$\therefore V$ and β have inverse effect on the medium freezing temperature T.

6.26



Let

$$\begin{aligned} N &= \text{total number of crystals in cloud} \\ m &= \text{mass of each crystal (in grams)} \\ \text{Total mass of crystals} &= mN \text{ grams} \\ \text{Total mass of liquid water (in gm)} &= 2 \times \underbrace{(10 \times 3) \times 10^9}_{\text{volume of cloud in } m^3} \end{aligned}$$

$$\therefore mN = 2 \times 30 \times 10^9 = 6 \times 10^{10} \text{ grams} \qquad \text{(A)}$$

But, since there are 1/liter of ice crystals in the cloud

$$N = \underbrace{(10 \times 3) \times 10^9}_{\text{volume of cloud in } m^3} \quad \underbrace{\left(1 \times 10^3\right)}_{\text{number of crystals per } m^3}$$

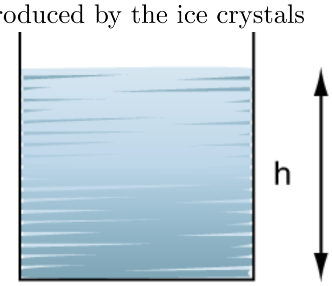
$$\underline{\boldsymbol{N = 3 \times 10^{13}}}$$

From (A)

$$\begin{aligned} m = \text{Mass of each ice crystal} &= \frac{\text{Total mass of crystals}}{N} \\ &= \frac{6 \times 10^{10}}{3 \times 10^{13}} \\ &= 2 \times 10^{-3} \text{ grams} \end{aligned}$$

$$\underline{\boldsymbol{m = 2 \text{ mg}}}$$

If h is rainfall (in cm) produced by the ice crystals

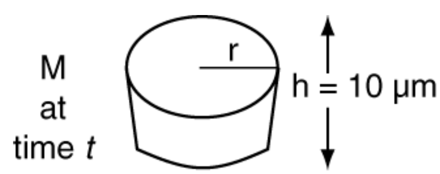


$$\underset{\text{cm}^3}{(Ah)}\ \underset{\text{g cm}^{-3}}{(\text{density of water})} = mN \text{ grams}$$

Since

$$\begin{aligned} A &= 10 \times 10^{10} cm^2 \\ \text{density of water} &= 1 \text{ g cm}^{-3} \\ 10^{11} h &= mN = 6 \times 10^{10} \\ \therefore h &= \underline{\mathbf{0.6\ cm = 6\ mm}} \end{aligned}$$

6.27



From (6.36)

$$\frac{dM}{dt} = \frac{C}{\varepsilon_o}(G_i S_i)$$

In SI units for a cylindrical disk of radius r

$$C = 8r\varepsilon_o$$

$$\therefore \frac{dM}{dt} = 8r(G_i S_i)$$

Also,

$$M = \pi r^2 h \rho_I$$

$$\therefore \frac{d}{dt}\left(\pi r^2 h \rho_I\right) = 8r G_i S_i$$

Since h and ρ_I are constant

$$\pi h \rho_I 2r \frac{dr}{dt} = 8rG_iS_i$$

or

$$\frac{dr}{dt} = \frac{4G_iS_i}{\pi h \rho_I}$$

At $-5°$C we see from Fig. 6.32, that

$$G_iS_i \simeq 2 \times 10^{-9} \text{ kg s}^{-1} \text{ m}^{-1}$$

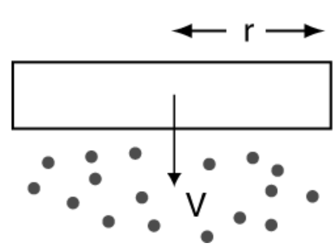
Since

$$\begin{aligned}
h &= 10 \ \mu\text{m} = 10^{-5} \text{ m} \\
\rho_I &= 0.917 \times 10^3 \text{ kg m}^{-3} \\
t &= 30 \times 60 = 1800 \text{ sec} \\
\therefore \int_o^r dr &= \frac{4\left(2 \times 10^{-9}\right)}{\pi\left(10^{-5}\right)(0.917)} \int_o^t dt \\
\therefore r &= \frac{8 \times 10^{-9}}{\pi 10^{-5}(0.917)\left(10^3\right)} t \\
&= \frac{8 \times 10^{-4}}{\pi(0.917)} 1800 \\
&= 4997 \times 10^{-7} \text{ m} \\
&= 0.0005 \text{ m}
\end{aligned}$$

$$\boldsymbol{r = 0.5 \text{ mm}}$$

$$\begin{aligned}
\text{Mass of the ice crystal} &= \pi r^2 h \rho_I \\
&= \pi\left(0.5 \times 10^{-3}\right)^2\left(10 \times 10^{-6}\right)\left(0.917 \times 10^3\right) \\
&= 0.72 \times 10^{-9} \text{ kg} \\
&= \underline{\mathbf{7.2} \ \boldsymbol{\mu}\mathbf{g}}
\end{aligned}$$

6.28



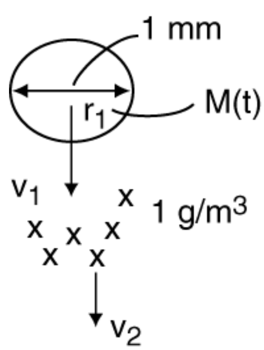
$$\begin{aligned}
\frac{dM}{dt} &= \pi r^2 E w v \\
&= \pi r^2 E w 2.4\left(M 10^6\right)^{0.24}
\end{aligned}$$

where M is in kg.

$$
\begin{aligned}
\therefore \int_{0.01\text{ mg}}^{0.05\text{ mg}} \frac{dM}{M^{0.24}} &= \int_o^t \pi r^2 Ew2.4\left(10^{1.44}\right) dt \\
\left[\frac{M^{0.76}}{0.76}\right]_{10^{-8}\text{ kg}}^{5\times10^{-8}\text{ kg}} &= \pi r^2 Ew2.4\left(10^{1.44}\right) t \\
\left[M^{0.76}\right]_{10^{-8}\text{ kg}}^{5\times10^{-8}\text{ kg}} &= 0.76\pi r^2 Ew2.4\left(10^{1.44}\right) t \\
\left(5\times10^{-8}\right)^{0.76} - \left(10^{-8}\right)^{0.76} &= 0.76\pi \underbrace{\left(0.5\times10^{-3}\right)^2}_{\substack{r\text{ in}\\ \text{meters}}} 0.1 \underbrace{\left(0.5\times10^{-3}\right)}_{\substack{w\text{ in}\\ \text{kg m}^{-3}}} 2.4\left(10^{1.44}\right) t \\
2.826\times10^{-6} - 8.31\times10^{-7} &= 0.43\times10^{-9}10^{1.44}t \\
28.26\times10^{-7} - 8.31\times10^{-7} &= 11.84\times10^{-9}t \\
\frac{19.95\times10^{-7}}{11.84\times10^{-9}} &= t \\
\therefore t &= 1.68\times10^{2}\text{ secs}
\end{aligned}
$$

$\underline{\boldsymbol{t = 2.8}\ \textbf{mins}}$

6.29



$$
\begin{aligned}
\frac{dM}{dt} &= \pi r_1^2 \left(V_1 - V_2\right) E\left(IWC\right) && \text{(A)} \\
M &= \frac{4}{3}\pi r_1^3 \rho_{\text{ice}} \\
\therefore\quad \frac{dM}{dt} &= \frac{4}{3}\pi r_1^2 \rho_{ice} \frac{dr_1}{dt} && \text{(B)}
\end{aligned}
$$

From (A) and (B)

$$\begin{aligned}
4\pi r_1^2 \rho_{\text{ice}} \frac{dr_1}{dt} &= \pi r_1^2 (V_1 - V_2) E (IWC) \\
\therefore \frac{dr_1}{dt} &= \frac{(V_1 - V_2) E (IWC)}{4\rho_{\text{ice}}} \\
\int_{0.5 \text{ mm } = 0.5\times 10^{-3} \text{ m}}^{0.5 \text{ cm } = 0.5\times 10^{-2} \text{ m}} dr_1 &= \frac{(V_1 - V_2) E (IWC)}{4\rho_{\text{ice}}} \int_o^t dt \\
0.5 \times 10^{-2} - 0.5 \times 10^{-3} &= \frac{1 (1) \left(10^{-3}\right)}{4 \times (100)} t \\
\therefore t &= \left(4.5 \times 10^{-3}\right) \frac{4 (100)}{10^{-3}} \\
&= 1800 \text{ secs}
\end{aligned}$$

$$\boxed{\boldsymbol{t = 30 \text{ mins}}}$$

6.30 Sufficient heat must be provided to evaporate fog droplets and to make the temperature of the air sufficiently to accommodate the additional water vapor.

$$\begin{aligned}
\text{Heat to evaporate fog droplets} &= \left(0.3 \times 10^{-3}\right) \left(2.477 \times 10^6\right) \\
&= 743 \text{ J}
\end{aligned}$$

Original saturated vapor pressure = 12.27 hPa. Corresponding density of water vapor ρ is given by

$$\begin{aligned}
12.27 \times 100 &= 461 \times 282 \times \rho \\
\therefore \rho &= 9.4 \times 10^{-3} \text{ kg m}^{-3}
\end{aligned}$$

Assitional amount of water vapor in air after evaporation of fog droplets

$$= \left(0.3 \times 10^{-3} \text{ kg}\right) \text{m}^{-3}$$

Total amount of water vapor after evaporation

$$\begin{aligned}
&= (0.3 + 9.4)\, 10^{-3} \text{ kg m}^{-3} \\
&= 9.7 \times 10^{-3} \text{ kg m}^{-3}
\end{aligned}$$

This density of vapor must correspond to the new saturated vapor pressure p at the required temperature T, therefore

$$p = 461 \left(9.7 \times 10^{-3}\right) T$$

or

$$p/T = 461 \times 9.7 \times 10^{-3}$$

or, if p is in hPa

$$\begin{aligned}
\frac{p100}{T} &= 461 \times 9.7 \times 10^{-3} \\
&= 0.045
\end{aligned}$$

or

$$\frac{p\,(\text{hPa})}{T\,(^\circ\text{K})} = 4.5 \times 10^{-4}$$

Inspect heat to find p/T with this value. Find,

$$T = 283.66^\circ\text{K} = 10.66^\circ\text{C}$$

Hence,

$$\text{Heat required to raise 1 m}^{-3}\text{ of air by } 0.66^\circ\text{C}$$

is

$$\begin{aligned}
&(\text{mass of air})\,(\text{specific heat of air})\,\Delta T \\
&= (1 \times \underbrace{1.275 \times \frac{273}{283}}_{\rho_{\text{air}} \times \frac{T_2}{T_1}})\left(1004\text{ J K}^{-1}\text{ kg}^{-1}\right) 0.66 \\
&= 815\text{ J}
\end{aligned}$$

$$\begin{aligned}
\therefore \text{Total heat needed} &= (743 + 815)\,\text{J/m}^3 \\
&= \underline{\mathbf{1558\ J}}
\end{aligned}$$

6.31

$$\begin{aligned}
\text{Number of drops} &= \frac{40 \times 10^{-3}\ \ (m^m)}{\text{Initial volume of each drop}} \\
&= \frac{40 \times 10^{-3}}{\frac{4}{3}\pi\,(0.25 \times 10^{-3})^3} \\
&= 6.1 \times 10^8
\end{aligned}$$

$$\begin{aligned}
\text{Final volume of water in drops} &= \frac{4}{3}\pi\left(2.5 \times 10^{-3}\right)^3\left(6.1 \times 10^8\right) \\
&= 39.9\text{ m}^3
\end{aligned}$$

$$\begin{aligned}
\text{Rainfall} &= \frac{39.9}{10 \times 10^6} m \\
&= \underline{\mathbf{4 \times 10^{-3}\ mm}}
\end{aligned}$$

6.32 Increase in mass of drops in Exercise 6.31

$$= \frac{(2.5)^3}{(0.25)^3} = \underline{10^3}$$

Increase in mass of drops in this exercise

$$
\begin{aligned}
&= \frac{(2.5)^3}{(20 \times 10^{-3})^3} \\
&= \underline{\mathbf{1.95 \times 10^6}} = \underline{\mathbf{2 \times 10^6}}
\end{aligned}
$$

6.33

$$
\begin{aligned}
R_S &= R_N \sqrt[3]{\left(\frac{N_N}{N_S}\right)} = 5 \sqrt[3]{\left(\frac{10}{10^4}\right)} mm \\
&= \underline{\mathbf{0.5\ mm}}
\end{aligned}
$$

6.34

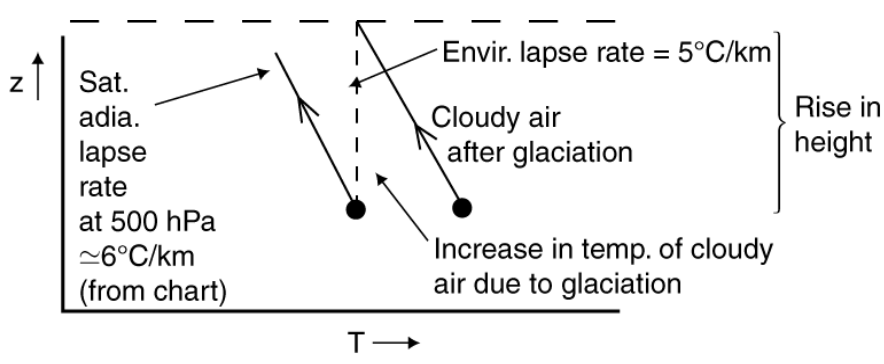
$$
\begin{aligned}
\text{Heat released by freezing} &= w_\ell 10^{-3} L_f \\
\text{Heat released due to condensation of excess water onto ice} &= (w_s - w_i)\, 10^{-3} L_d
\end{aligned}
$$

$$\therefore \boldsymbol{c\Delta T =} \underline{\boldsymbol{w_\ell 10^{-3} L_f + (w_s - w_i)\, 10^{-3} L_d}}$$

6.35

$$
\begin{aligned}
c\Delta T &= w_\ell 10^{-3} L_f \\
(1004)\, \Delta T &= \left(2 \times 10^{-3}\right) \left(3.34 \times 10^5\right) \\
\Delta T &= \frac{2 \times 10^2 \times 3.34}{1004} \\
&= \underline{\mathbf{0.7^\circ C}}
\end{aligned}
$$

6.36



$$\text{Incremental rise in cloudy air} = \frac{\begin{array}{c}\text{Increase in temperate due to}\\ \text{freezing of cloud water}\end{array}}{\begin{array}{c}\text{Difference in lapse rates}\\ (\simeq 1^\circ\text{C/km})\end{array}} \qquad (1)$$

$$\text{Heat released by glaciation per m}^3\text{ of air} = \left(0.001\text{ kg m}^{-3}\right)\left(3.34\times10^5\text{ J kg}^{-1}\right) = 3.34\times10^2\text{ J m}^{-3}$$

If $\rho_{500\text{ hPa}}$ is density of air (in kg m^{-3}) at 500 hPa and -20°C, 1 m^3 of air at 500 hPa has mass $\rho_{500\text{ hPa}}$ kg.

$$\therefore \text{Heat released per kg of cloudy air} = \frac{3.34\times10^2}{\rho_{500\text{ hPa}}}\text{ J kg}^{-1} \qquad (2)$$

Since $\rho \propto p/T$

$$\begin{aligned}
\frac{\rho_{500\text{ hPa and 253 K}}}{\rho_{1000\text{ hPa and 273 K}}} &= \frac{500/253}{1000/273} = \frac{1}{2}\frac{273}{253} \\
\therefore \rho_{500\text{ hPa and 253 K}} &= 0.54\ \rho_{1000\text{ hPa and 273 K}} \\
&= 0.54\,(1.275)\ \text{kg m}^{-3} \\
&= 0.688\text{ kg m}^{-3} \qquad (3)
\end{aligned}$$

From (2) and (3),

$$\begin{aligned}
\text{Heat released per kg of cloudy air} &= \frac{3.34\times10^2}{0.688} \\
&= 485\text{ J}
\end{aligned}$$

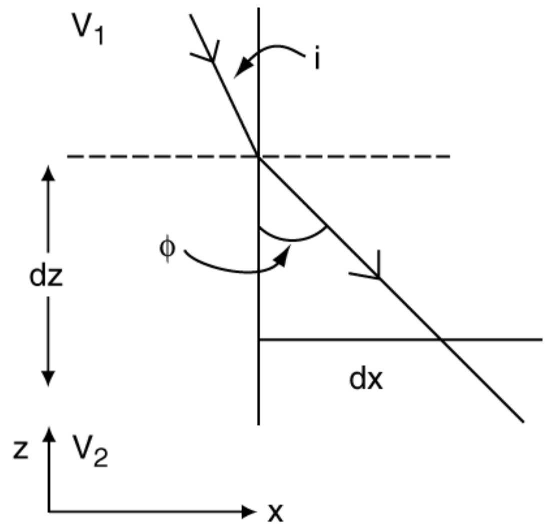
If temperature rise of cloudy air due to glaciation is ΔT°C

$$\begin{aligned}
(1\text{ kg})\,(\Delta T)\left(c_p\text{ J kg}^{-1}\ {}^\circ\text{C}^{-1}\right) &= 485\text{ J} \\
\therefore \Delta T &= \frac{485}{1004}{}^\circ\text{C} = \underline{\mathbf{0.48^\circ C}} \qquad (4)
\end{aligned}$$

From (1) and (4)

$$\begin{aligned}
\text{Incremental rise in cloudy air} &= \frac{0.48^\circ\text{C}}{1^\circ\text{C km}^{-1}} \\
&= 0.48\text{ km} \\
&= 480\text{ m}
\end{aligned}$$

6.37



From the law of refraction

$$\frac{\sin i}{\sin r} = \frac{V_1}{V_2} \tag{1}$$

Or, in general,

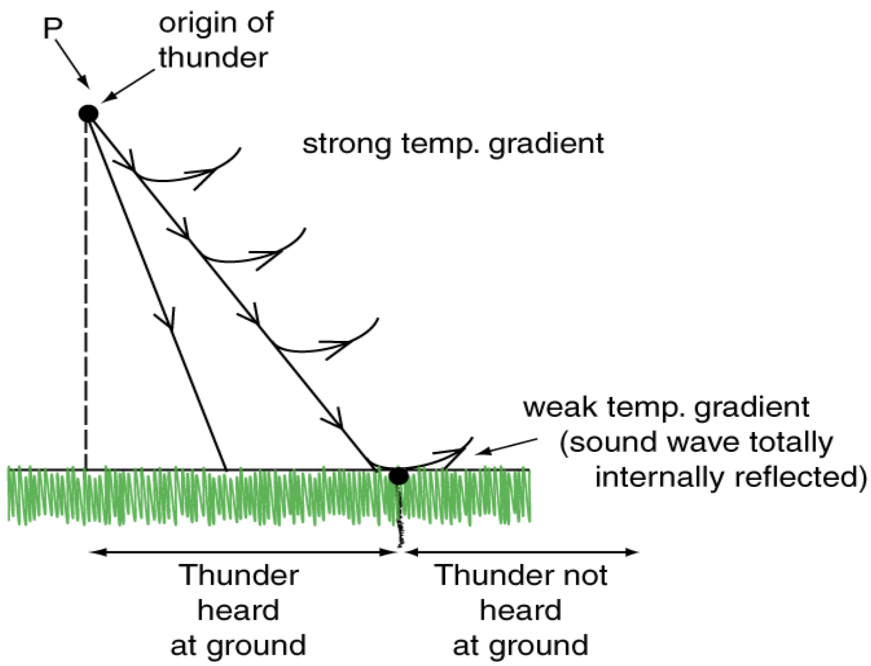
$$\frac{\sin \phi}{V} = \text{constant (along any path of a sound wave)} \tag{2}$$

But,

$$\tan \Phi = \left|\frac{dx}{dz}\right| = -\frac{dx}{dz}$$

$$\therefore \frac{\sin \Phi}{\cos \Phi} = \frac{\sin \Phi}{\left(1 - \sin^2 \Phi\right)^{1/2}} \tag{3}$$

From (2) at point P in the diagram below:



From (2) at point P in above diagram

$$\frac{\sin 90^\circ}{V_o} = \text{constant}$$

or,

$$\frac{1}{V_o} = \text{constant} \tag{4}$$

where, V_o is velocity of sound just above ground level. Hence, in general, from (2)

$$\frac{\sin \Phi}{V} = \frac{1}{V_o}$$

But $V \propto \sqrt{T}$, therefore,

$$\sin \Phi = \frac{V}{V_o} = \sqrt{\frac{T}{T_o}} \tag{5}$$

From (3) and (5)

$$-\frac{dx}{dz} = \frac{\sqrt{(T/T_o)}}{\sqrt{\left(1 - \frac{T}{T_o}\right)}} = \sqrt{\frac{T}{T_o - T}}$$

But,

$$\begin{aligned} T &= T_o - \Gamma z \\ \therefore T_o - T &= \Gamma z \end{aligned}$$

and,

$$\boldsymbol{dx = -\sqrt{\left(\frac{T}{\Gamma z}\right)}dz}$$

6.38 From Exercise 3.37:

$$\begin{aligned}
dx &= -\left(\frac{T_o - \Gamma z}{\Gamma z}\right)^{1/2} dz \\
&= -\left(\frac{T_o}{\Gamma z} - 1\right)^{1/2} dz \\
&= -\left(\frac{T_o}{\Gamma z}\right)^{1/2}\left(1 - \frac{\Gamma z}{T_o}\right)^{1/2} dz \\
&= -\left(\frac{T_o}{\Gamma z}\right)^{1/2}\left(1 - \frac{1}{2}\frac{\Gamma z}{T_o} + \begin{array}{c}\text{higher powered}\\ \text{terms in z}\end{array}\right) dz \\
dz &\simeq -\left(\frac{T_o}{\Gamma z}\right)^{1/2}\left(1 - \frac{1}{2}\frac{\Gamma z}{T_o}\right) dz
\end{aligned}$$

Integrating

$$\begin{aligned}
\int_o^D dx &= -\sqrt{\frac{T_o}{\Gamma}}\int_H^o \left(\frac{1}{\sqrt{z}} - \frac{1}{2}\frac{\Gamma z}{T_o}\right) dz \\
&= -\left(\frac{T_o}{\Gamma}\right)^{1/2}\int_H^o \left(\frac{1}{\sqrt{z}} - \frac{1}{2}\frac{\Gamma z}{T_o}\right) dz \\
&= -\left(\frac{T_o}{\Gamma}\right)^{1/2}\left[2z^{1/2} - \frac{\Gamma}{2T_o}\frac{2}{3}z^{3/2}\right]_H^o \\
&= -\left(\frac{T_o}{\Gamma}\right)^{1/2}\left[2z^{1/2} - \frac{\Gamma}{3T_o}z^{3/2}\right]_H^o \\
&= -\left(\frac{T_o}{\Gamma}\right)^{1/2}\left[-2H^{1/2} + \underbrace{\frac{1}{3}\frac{\Gamma}{T_o}H^{3/2}}_{\text{small term}}\right]
\end{aligned}$$

$$\boxed{\boldsymbol{D = 2\left(\frac{T_o H}{\Gamma}\right)^{1/2}}}$$

with $\Gamma = 7.5^\circ\ \mathrm{km}^{-1}$ and $T_o = 300$ K, $H = 4$ km

$$D = 2\left(\frac{300 \times 4000}{7.5 \times 10^{-3}}\right)^{1/2} = 2\left(1.26 \times 10^4\right)$$

$$\boxed{\boldsymbol{D = 25.3\ \mathrm{km}}}$$

6.39

$$\begin{aligned}\text{Velocity} &= \frac{\text{Distance}}{\text{Time}}\\ \therefore \text{Distance} &= (\text{velocity})\,(\text{time})\\ &= \left(0.34 \text{ km s}^{-1}\right)(10)\\ &= \underline{\mathbf{3.4\ km}}\end{aligned}$$

$$\begin{aligned}\text{Minimum length of flash} &= (8 \text{ secs})\left(0.34 \text{ km sec}^{-1}\right)\\ &= \underline{\mathbf{2.72\ km}}\end{aligned}$$

This represents true length of flash only if lightning stroke is aligned along site of observer.

6.40 Solution:

(a) The oxidation number of hydrogen in most of its compounds is +1 and the oxidation number of oxygen in most of its compounds is −2. Hence, if x represents the oxidation number of sulfur in $HSO_3^-(aq)$, and since the net charge on $HSO_3^-(aq)$ is −1,

$$+1 + x + 3(-2) = -1$$

therefore,

$$x = 4$$

That is, the oxidation number of sulfur in $HSO_3^-(aq)$ is 4. For this reason, sulfur in $HSO_3^-(aq)$ is often referred to as "sulfur four" or S(IV).

Similarly, if y is the oxidation number of sulfur in $H_2SO_4(aq)$, since there is no net charge on $H_2SO_4(aq)$,

$$2(+1) + y + 4(-2) = 0$$

therefore,

$$y = 6$$

Hence, the sulfur in $H_2SO_4(aq)$ has an oxidation number of 6 ("sulfur six" or S(VI)). Therefore, when $H_2SO_3^-(aq)$ is converted to $H_2SO_4(aq)$, the oxidation number of sulfur increases from 4 to 6.

(b) By following similar steps the reader can show that the oxidation number of sulfur in both $SO_2 \cdot H_2O(aq)$ and in $SO_3^{2-}(aq)$ is 4. Therefore, when these species are converted to H_2SO_4 their oxidation numbers also increase by 2.